On the Nondomination of Cohorts Coteries

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Abstract

In this paper, we show that a subset of *Cohorts coteries* proposed in [5] are *nondominated (ND)* k-coteries, which are candidates to achieve the highest availability when utilized to solve the distributed k-mutual exclusion problem and the distributed h-out of-k mutual exclusion problem.

Index Terms — *Availability*, *k-coteries*, *mutual exclusion*, *nondomination*, *quorums*

1 Introduction

In [5], Jiang et al. proposed ***Cohorts structures*** to aid the construction of a class of *k-coteries* called ***Cohorts coteries***, where a Cohorts structure $Coh(k,n)=(C_1,...,C_n)$ is a list of sets (each called a ***cohort***) satisfying:

P1. $|C_1|=k$.
P2. $\forall i : 1< i \leq n : |C_i| > max(2k-2, k)$.
P3. $\forall i, j$: $1\leq i, j\leq n, i\neq j$: $C_i \cap C_j=\emptyset$.

In an earlier paper [3], Huang et al. constructed Cohorts coteries with P1 being relaxed to be $|C_1|\geq k$. In [8], Neilsen and Mizuno showed that a Cohorts coterie may be *dominated* if P1 is relaxed to be $|C_1|\geq k$. However, it is left open whether a Cohorts coterie is dominated or not when P1 assumes $|C_1|=k$. In this paper, we show that if P1 assumes $|C_1|=k$, then a Cohorts coterie is a ***nondominated*** *(**ND**)* k-coterie.

A *k*-coterie [1, 3] is a family of sets called *quorums* satisfying the *intersection property*: there are at most *k* pairwise disjoint quorums. It can be used to develop the distributed *k-mutual exclusion* algorithm [7] and the distributed *h-out of-k mutual exclusion* algorithm [6]. The basic idea of such algorithms is simple: a node should collect permissions from nodes of a quorum (resp. *h* pairwise disjoint quorums, $1 \le h \le k$) to gain one entry (resp. *h* entries) to a *critical section (CS)*. Since a node grants its permission to one node at a time, the intersection property then guarantees that at most *k* entries to the CS are admitted simultaneously. The algorithms using *k*-coteries usually incur low message cost and can tolerate node and/or network link failures, even when the failures lead to network partitioning. For any specific value of *h* ($1 \le h \le k$), ND *k*-coteries are candidates to endow the algorithms with the highest *availability*, which is the probability that at least *h* entries to a *critical section* are available in an error-prone environment. Thus, we should always concentrate on ND *k*-coteries when availability is a significant concern.

There are two definitions of *k*-coteries. Fujita, Yamashita and Ae first proposed the definition of *k*-coteries in [1]; Huang, Jiang and Kuo proposed another definition in [3] independently. In [9], Neilsen and Mizuno regarded *k*-coteries as those defined by Huang et al. and used the term "*proper k-coteries*" to refer to those defined by Fujita et al. In [2], Harada and Yamashita regarded *k*-coteries as those defined by Fujita et al. and used the term "*k-semicoteries*" to refer to those defined by Huang et al. In this paper, we adopt Harada and

Yamashita's way to differentiate the two k-coterie definitions. To be more precise, a k-coterie holds the non-intersection property, while a k-semicoterie does not. The non-intersection property guarantees that for any h ($h \le k$) pairwise disjoint quorums, there must exist a quorum Q such that Q and the h quorums are also pairwise disjoint. Thus, when applied to design k-mutual exclusion algorithms (or h-out of-k mutual exclusion algorithms), k-coteries always achieve higher degree of concurrency than k-semicoteries.

2 Nondomination of Cohorts Coteries

A *k-semicoterie* $\mathcal{C}$ under universal set U is a family of subsets of U. Each member in $\mathcal{C}$ is called a *quorum* and should observe the following two properties [3]:

Minimality Property: Any quorum is not a super set of another quorum.

Intersection Property: There are at most k pairwise disjoint quorums.

A k-semicoterie is also a k-coterie [1] if it further satisfies the following non-intersection property:

Non-intersection Property: For any h ($h<k$) pairwise disjoint quorums $Q_1,...,Q_h$, there exists a quorum Q_{h+1} such that $Q_1,...,Q_{h+1}$ are pairwise disjoint.

In [5], Cohorts structures are used to help construct k-coteries. Given a Cohorts structure $Coh(k,n)=(C_1,\ldots,C_n)$, a set Q is said to be a ***quorum under Coh(k, n)*** if some cohort C_i is Q's ***primary cohort***, and each cohort C_j, $j > i$, is a ***supporting cohort*** of Q, where a cohort C is Q's primary cohort if $|Q\cap C|=|C|-(k-1)$ (i.e., Q contains exactly all except $k-1$ members of C), and a cohort C is a supporting cohort of Q if $|Q\cap C|=1$ (i.e., Q contains exactly one member of C). The family of quorums under $Coh(k, n)$ is called a Cohorts coterie, which has been shown to be a k-coterie in [5].

Let $\mathcal{C}$ and $\mathcal{D}$ be two distinct k-coteries (or k-semicoteries). $\mathcal{C}$ is said to *dominate* $\mathcal{D}$ if and only if every quorum in $\mathcal{D}$ is a super set of some quorum in $\mathcal{C}$ (i.e.,$\forall Q$, $\exists Q'$: $Q\in\mathcal{D}$, $Q'\in\mathcal{C}$: $Q'\subseteq Q$). Obviously, the dominating one ($\mathcal{C}$) has more chances than the dominated one ($\mathcal{D}$) to have *available quorums* in an error-prone environment, where a quorum is said to be *available* if all of its members (nodes) are *up*. Note that an available quorum implies an available entry to the CS. Thus, when availability is a significant concern, we should always concentrate on ND k-coteries (or k-semicoteries) that no other k-coterie (or k-semicoterie) can dominate.

Below, we show that Cohorts coteries are ND k-coteries (with P1 being $|C_1|=k$) on the basis of Theorem 1, which was proposed in [4] and [8] simultaneously, and was restated in [2].

Theorem 1 ([2, 4, 8]). Let $\mathcal{C}$ be a k-semicoterie under universal set U. $\mathcal{C}$ is dominated if and only if there exists a set $X\subseteq U$ such that

A1. For any quorum $Q\in\mathcal{C}$, $Q\not\subseteq X$.

A2. For any k pairwise disjoint quorums $Q_1,...,Q_k\in\mathcal{C}$, there exists an i, $1\le i\le k$, such that $Q_i\cap X\neq\emptyset$.

Theorem 2. Let $\mathcal{C}$ be a family of quorums under $Coh(k, n)=(C_1,...,C_n)$, $n\geq 1$. $\mathcal{C}$ is an ND k-coterie.

Proof: As shown in [5], $\mathcal{C}$ is a k-coterie; it is thus a k-semicoterie. Below, we first prove that $\mathcal{C}$ is an ND k-semicoterie. The proof is by induction on the value of n.

Basis: $n=1$.

Let $C_1=\{u_1,...,u_k\}$ (by P1, $|C_1|=k$). Then, the family of all the quorums under $Coh(k,1)=(C_1)$ is $\{ \{u_1\},...,\{u_k\} \}$, which is a k-singleton coterie and is shown to be ND in [4].

Induction Hypothesis: Assume the family of quorums under $Coh(k, n-1)=(C_1,\dots,C_{n-1})$ is ND.

Induction Step: On the basis of the induction hypothesis, we want to prove that $\mathcal{C}$ is ND.

The proof is done by contradiction. Suppose that the family $\mathcal{C}$ of quorums under $Coh(k, n)$ is dominated, then by Theorem 1, we can find a set X satisfying

A1. For any quorum Q under $Coh(k, n)$, $Q \not\subseteq X$.

A2. For any k pairwise disjoint quorums $Q_1,...,Q_k$ under $Coh(k, n)$, there exists an i, $1\leq i\leq k$, such that $Q_i \cap X \neq \emptyset$.

Let $C_n=\{v_1,...,v_s\}$, where $s=|C_n|>max(2k-2, k)$ (by P2). Then, by definition, a quorum under $Coh(k, n)$ may take C_n as the primary cohort with no supporting cohort, or may take C_m, $1\leq m\leq n-1$, as the primary cohort with $C_{m+1},\dots,C_n$ being supporting cohorts. Thus, a quorum under $Coh(k, n)$ may be of the form: either (**form-1**) a set of $s-(k-1)$ members of C_n, or (**form-2**) a quorum under $Coh(k, n-1) \cup \{v_j\}$, $1\leq j\leq s$.

Let Q be a form-1 quorum. By A1, we have $Q \not\subseteq X$. It follows that X should have less than $s-(k-1)$ members of C_n, i.e., C_n has at least k members not in X. Without loss of generality, let $v_1,\ldots,v_k$ be the k members of C_n that are not in X.

Let $Q_1',\ldots,Q_k'$ be k pairwise disjoint quorums under $Coh(k, n-1)=(C_1,\ldots,C_{n-1})$ (P1, P2 and P3 ensure the existence of the k quorums). Then, $Q_1=Q_1'\cup\{v_1\},\ldots,Q_k=Q_k'\cup\{v_k\}$ are k pairwise disjoint form-2 quorums under $Coh(k, n)$. By A2, there exists an i, $1\leq i\leq k$, such that $Q_i\cap X\neq\emptyset$. Since $v_1,\ldots,v_k$ are not in X, we have that there exists an i, $1\leq i\leq k$, such that $Q_i'\cap X\neq\emptyset$. Because $Q_1',\ldots,Q_k'$ contains no member of C_n (by P3), we infer that there exists an i, $1\leq i\leq k$, such that $Q_i'\cap(X-C_n)\neq\emptyset$. We have

A2′. For any k pairwise disjoint quorums $Q_1',\ldots,Q_k'$ under $Coh(k, n-1)$, there exists an i, $1\leq i\leq k$, such that $Q_i'\cap(X-C_n)\neq\emptyset$.

Now, suppose there exists a quorum Q' under $Coh(k, n-1)$ such that $Q'\subseteq(X-C_n)$. Then, we have $(Q'\cup\{v_j\}) \subseteq X$ for $j=k+1,\ldots,s$. This contradicts A1 because $Q'\cup\{v_j\}$ is a form-2 quorum under $Coh(k, n)$. Thus, we have

A1′. For any quorum Q' under $Coh(k, n-1)$, $Q' \not\subseteq (X-C_n)$.

By A1′ and A2′, we have that the family of quorums under $Coh(k, n-1)$ is dominated, which contradicts the induction hypothesis. So, the family $\mathcal{C}$ of quorums under $Coh(k, n)$ must not be dominated. It is hence ND.

Thus, by the induction principle, $\mathcal{C}$ is an ND k-semicoterie for any n, $n\geq 1$. As noted in [2], any ND k-semicoterie is an ND k-coterie if it satisfies the non-intersection property. In [5], $\mathcal{C}$ has been shown to be a k-coterie, which satisfies the non-intersection property. Hence, $\mathcal{C}$ is an ND k-coterie. ■

3 Conclusion

The k-coterie can be utilized to design distributed k-mutual exclusion algorithms and distributed h-out of-k mutual exclusion algorithms. The k-coterie-based algorithms usually incur low message cost and have high availability. They can tolerate node and/or network link failures, even when the failures lead to network partitioning. In this paper, we have shown that a subset of Cohorts coteries proposed in [5] are ND k-coteries, which are candidates to endow the k-coterie-based algorithms with the highest availability.

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Jehn-Ruey Jiang received his Ph. D. degree in Computer Science in 1995 from National Tsing-Hua University, Taiwan. He joined Chung-Yuan Christian University as an Associate Professor in 1995. He is currently an Associate Professor of the Department of Information Management, Hsuan-Chuang University. He is a recipient of the Best Paper Award in Int'l Conf. on Parallel Processing, 2003. His research interests include distributed algorithms, distributed computing, distributed fault-tolerance, mobile computing, protocols for mobile ad hoc networks and wireless sensor networks.