Broadcasting with Optimized Transmission Efficiency in 3-Dimensional Wireless Networks

Yung-Liang Lai and Jehn-Ruey Jiang
Department of Computer Sciences and Information Engineering
National Central University, Taiwan
calvinlai@axp1.csie.ncu.edu.tw, jrjiang@csie.ncu.edu.tw

Abstract

Broadcasting is one of the most important operations in the wireless network for disseminating information throughout the entire network. Flooding is a simple mechanism to realize broadcasting, but it has high redundancy of retransmissions, leading to low transmission efficiency. Many broadcast protocols have been proposed for pursuing optimized transmission efficiency for wireless networks hypothetically deployed on the 2-dimesional (2D) plane. In the real world, wireless networks are deployed in the 3D space. In this paper, we derive the upper bound of 3D transmission efficiency and propose a 3D broadcast protocol with optimized transmission efficiency by partitioning the 3D space into multi-layer hexagonal prisms of a hexagon ring pattern in each layer. As we will show, the transmission efficiency of the proposed protocol can reach 1/π, which is better than those of other polyhedron-filling approaches using cubes, hexagon prisms, rhombic dodecahedrons, and truncated octahedrons.

1. Introduction

Broadcasting is one of the most important operations in the wireless network for disseminating information throughout the entire network. The operation has many applications; for example, many routing protocols rely on broadcasting packets to find routing paths to a destination node. Flooding is an intuitive approach to realize broadcasting, in which each node retransmits a packet when receiving it for the first time. Flooding is simple and is highly reliable; however, it may cause the broadcast storm problem [6] and has low *transmission efficiency* due to redundancy of retransmissions. As shown in [7], the theoretical upper bound of transmission efficiency is 0.61 for wireless networks deployed on the 2-dimesional (2D) plane.

Some geometry-based broadcast protocols [5, 7, 10, 12] for 2D wireless networks have been proposed to pursue optimized transmission efficiency. Among them, the Optimized Broadcast Protocol (OBP) proposed in [12] achieves the highest transmission efficiency 0.55, which is about 90% of the theoretical upper bound. In OBP, a node relies on hexagon rings centered at the broadcasting source node to decide if it should retransmit a broadcast packet when it receives the packet for the first time.

In some real world cases, the wireless network is deployed in the 3-dimesional (3D) space. For example, in an underwater acoustic network, an air to air communication network, or a wireless network consisting of nodes distributed in different floors of a multi-storey building, the relationship between wireless communicating nodes should be 3D rather than 2D. This motivates us to investigate broadcasting and the transmission efficiency for 3D wireless networks.

Some papers [1-3, 11] study the arrangement of nodes to fully cover a given space with optimized number of nodes. Most of them [1-3] are based on the regular polyhedron filling approach, which partitions the space into 3D cells, each being a polyhedron located by a node in the center. The polyhedrons used to fill the space are cubes, hexagonal prisms, rhombic dodecahedrons, and truncated octahedrons. One paper [11] proposes using the body-center cubic (BCC) to fill the space; as shown in [1], filling space by BCCs is equivalent to filling space by truncated octahedrons, though.

When the center nodes of neighboring cells can communicate with each other, the polyhedron filling approach can be transformed into a 3D broadcast protocol by demanding the center node in each cell to retransmitting the broadcast packet. The papers [4] and [9] adopt a different strategy for designing 3D broadcast protocols; they demand vertex nodes, instead of center nodes, in hexagonal prisms to retransmit the broadcast packet for better transmission efficiency.

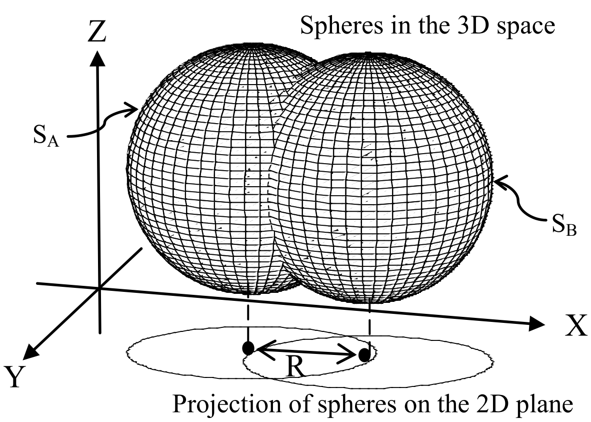


Fig. 1 Illustration of the upper bound of transmission efficiency in the 3D space

In this paper, we first generalize the definition of transmission efficiency for 3D wireless networks. To take two communicating nodes A and B in Fig. 1 as an example, the transmission efficiency is the ratio of the effective communication area (the region covered by spheres S_A or S_B, i.e., $|S_A \cup S_B|$) over the total communication region (the summation of volumes of S_A and S_B, i.e., $|S_A|+|S_B|$), where S_A and S_B are the spheres centered respectively at A and B with the radius of R, the transmission range. When the distance between nodes A and B equals to the transmission range R, transmission efficiency reaches the upper bound. As we will show, the theoretical upper bound is 0.84 in the 3D space.

We further propose an optimized broadcast protocol, called *3D optimized broadcast protocol (3DOBP)*, by partitioning the 3D space into multi-layer hexagonal prisms. As shown in Fig. 2(a), the network space is partitioned into multiple layers along the vertical axis (i.e., Z axis). And each layer consists of a set of cells each of which is a hexagonal prism, as shown in Fig. 2(b). From the top-to-down view, the cells in a layer form hexagon rings. As shown in Fig. 2(c), the hexagon rings of the middle layer is centered at the *source node* S (represented as ◎) that initiates the broadcast of a packet. From the source node and along the vertical axis, there is a *starting node* nearest to the center of a cell for each upper or lower layer. Only the starting nodes, the *vertex nodes* nearest to hexagon centers (represented as ●) and specific *vertex nodes* nearest to some hexagon vertexes (represented as ○) need to retransmit the packet. As we will show, the transmission efficiency of the proposed protocol can reach $1/\pi$. Compared with other polyhedron-filling approaches using cubes, hexagon prisms, rhombic dodecahedrons and truncated octahedrons, our proposed protocol is with the highest transmission efficiency.

The rest of this paper is organized as follows. In Section 2, we introduce some related work. In Section 3, we present the proposed protocol. In Section 4, we analyze transmission efficiency of the proposed protocol and compare the analyzed result with those of polyhedron-filling based approaches. And finally, some concluding remarks are drawn in Section 5.

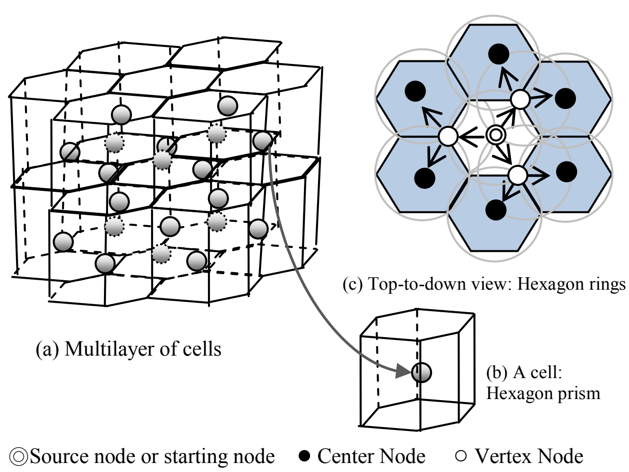


Fig. 2 Illustration of partitioning the 3D space into multi-layer hexagon prisms of the hexagon ring pattern

2. Related Work

The papers [2] and [3] study the base station arrangement problem for 3D picocellular networks. They rely on the hexagonal prism pattern and the rhombic dodecahedron pattern, respectively, for positioning base stations. The paper [1] studies the sensor deployment problem for 3D wireless sensor networks. The problem is to select the minimum number of sensors for covering a given space, while keeping sensors connected. The solution can be solved on the basis of Kelvin's conjecture. In 1887, Kelvin raised a question: "What is the optimal way to fill a three dimensional space with cells of equal volume, so that the surface area (interface area) is minimized?" Kelvin merely conjectured (but did not proved) that using truncated octahedrons to fill the space is optimal. In [1], truncated octahedrons are used for 3D sensor network deployment. Moreover, cubes, hexagonal prisms and rhombic dodecahedrons are also applied to the deployment for the sake of comparison. The structures are analyzed in terms of the number of cells needed to cover a given space. By the analysis, the truncated octahedron indeed presents the minimum number of nodes for deployment. The paper [11] proposes using the body-center cubic (BCC) structure for 3D sensor network deployment. By BCC, sensors are deployed at eight vertexes of a cube and at the center of the cube. As shown in [1], the BCC-based deployment is equivalent to truncated octahedron-based deployment by the Voronoi tessellation analysis.

The paper [4] proposes a 3D broadcast protocol for air to air communication. The main idea of the protocol is to partition the space into multiple planes, each of which is further partitioned according to the regular hexagon patter, as shown in Fig. 3 (a). Only the nodes at (or nearest to) hexagon vertexes need to retransmit the broadcast packet. To take the scenario in Fig. 3 (a) for an example, a source node (S) initially issues the broadcast packet destined to locations (1), (2), and (3). The node nearest to the location (1) is responsible for retransmitting the packet. Then, the node nearest to the location (11) is responsible for the retransmission. The nodes nearest to the locations (111) and (112) are then responsible for the retransmission. Similarly, the other nodes nearest to the locations of hexagon vertexes are responsible for the retransmission and hence one layer of a plane is fully covered. As shown in Fig. 3(b), the source node S also triggers a forward node on each upper or lower plane to start the retransmission processes. Consequently, all nodes in the space can receive the broadcast packet. Note that the paper [9] also reports a similar protocol.

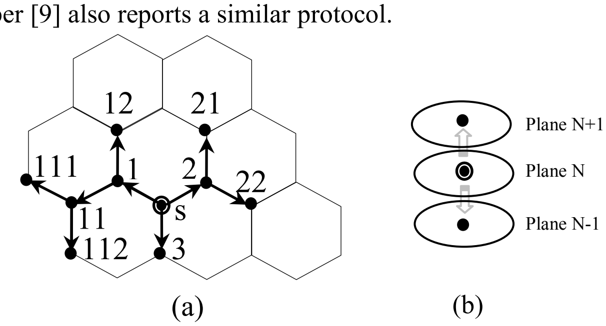


Fig. 3 Illustration of the broadcast protocol in [4, 9]

3. The Proposed Protocol

In this section, we describe the proposed protocol, 3-Dimensional Optimized Broadcast Protocol (3DOBP). We assume that each node is aware of its location but is not aware of the network topology. We also assume that all nodes are equipped with omnidirectional antennas, whose transmission ranges are of the same value R and are modeled as spheres of the radius R.

3.1. Overview of 3DOBP

In 3DOBP, the entire network space is divided into multiple layers of height H. Each layer consists of a set of cells, as shown in Fig. 4(a). As shown in Fig. 4(b), each cell is a hexagonal prism of the side length of L, which equals to the transmission range R. Fig. 4(c) shows that the *source node* or the *initial starting node* (represented as ◎) is in the middle layer (layer 0) to broadcast a packet and the *interlayer nodes* (represented as ⊗) is between layers to forward the packet to other *starting nodes* (also represented as ◎) in different layers (layers 1, -1, …, etc.) At a specific layer, only the nodes nearest to hexagon centers (represented as ●) and the nodes nearest to some specific hexagon vertexes (represented as ◯) need to retransmit the packet. Note that below we use the term *center node* (resp., *vertex node*) to stand for the node nearest to a hexagon center (resp., vertex).

We assume there is a node with transmission radius R located at the center of a hexagonal prism of side length L and height H. We can draw a circumsphere of radius Rc for the hexagonal prism. The relationship of L, H and Rc is $Rc = \sqrt{L^2 + H^2/4}$. We set V-Ratio to represent ratio of the volume of the hexagonal prism over the volume of the circumsphere as follows.

$$\text{V-Ratio} = \frac{3\sqrt{3}}{2} L^2 H / \left(\frac{4\pi}{3}\left(\sqrt{L^2 + \frac{H^2}{4}}\right)^3\right) \quad (1)$$

When V-Ratio is maximized, the volume of the hexagonal prism is maximized. So, we have to derive the first derivative of V-Ratio equation, and then find its solution. After some basic calculations, we have the solution

$$H = \sqrt{2}\, L \quad (2)$$

We set the transmission radius R to be the circumsphere radius Rc. We have

$$R = Rc = L\sqrt{3/2} \quad (3)$$

$$L = \frac{R}{\sqrt{3/2}} \quad (4)$$

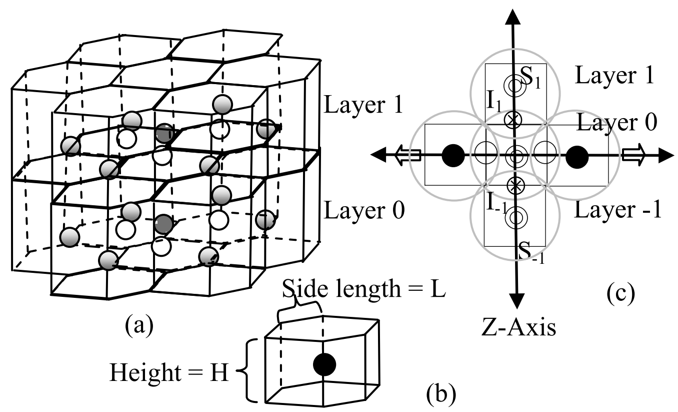
$$H = L\sqrt{2} = \frac{R}{\sqrt{3/2}}\sqrt{2} = 2\frac{R}{\sqrt{3}} \quad (5)$$

To sum up, we can use hexagonal prisms, each of which is of side length $\frac{R}{\sqrt{3/2}}$ and height $2\frac{R}{\sqrt{3}}$ to represent the *effective transmission space* of a node of transmission radius R, to fully cover the network space.

To broadcast a packet throughout the entire network space, we need to achieve the following two goals: (a) to activate starting nodes for retransmission in every layer along the Z-axis, and (b) to activate center nodes in every layer for retransmission. Below, we show how to achieve the two goals.

How to activate staring nodes in different layers

We partition the network space into multiple layers. The source node or the initial starting node S_0, which initiates the broadcast of a packet, is assumed to be in the layer 0. S_0 is required to activate the starting node S_t in every layer t with the help of interlayer nodes. S_0 just transmits the packet to its two neighboring interlayer nodes I_1 and I_{-1} in layer 1 and layer -1, respectively. Then, I_1 will forward the packet to starting node S_1 in layer 1 to activate it. Similarly, I_{-1} will forward the packet to the starting node S_{-1} in layer -1 to activate it. Likewise, stating nodes S_1 (or S_{-1}) will active starting node S_2 (or S_{-2}), and so on. So, the starting node in every layer will be activated along the Z-axis.



◎Starting node ● Center node ◯ Vertex node ⊗ Interlayer node

Fig. 4 Illustration of space partitioning of 3DOBP

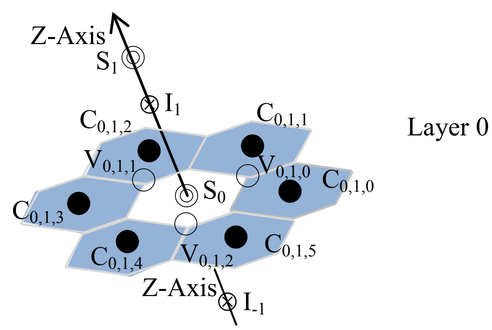


Fig.5 Illustration of activation of starting nodes along the Z-axis and activation of center nodes in the layer 0

How to activate center nodes in a layer

As shown in Fig. 5, each layer forms hexagonal rings that can fully cover a layer when the rings expand. Hence, if all center nodes are activated to forward the broadcast packet, then all the nodes in the layer can receive the broadcast packet. Note that not all vertex nodes need to be

activated. They are activated not for the purpose of covering the layer but for the purpose of forwarding the broadcast packet to center nodes. As we will show, a center node can rely on a vertex node to deliver packets to two center nodes. Thus, the number of activated vertex nodes is just half the number of activated center nodes.
As shown in Fig. 5, the source node S_0 in layer 0 first activates three vertex nodes $V_{0,1,0}$, $V_{0,1,1}$, $V_{0,1,2}$ and two interlayer nodes I_1, I_{-1}. One vertex node then activates two center nodes. For example, the vertex node $V_{0,1,0}$ activates two center nodes $C_{0,1,0}$ and $C_{0,1,1}$. Similarly, the vertex nodes $V_{0,1,1}$ and $V_{0,1,2}$ activate center nodes ($C_{0,1,2}$, $C_{0,1,3}$) and ($C_{0,1,4}$, $C_{0,1,5}$), respectively. We have mentioned how S_0 activates other starting nodes in different layers with the help of interlayer nodes. Below, we describe how S_0 activates all center nodes in a layer with the help of hexagon rings.

The hexagon rings have only one hexagon in the central (level-0) ring, and have six hexagons in the level-1 ring, and so on. In general, there are $6k$ hexagons in the level-k ring. A hexagon center in layer t and in the level-k ring is denoted as $C_{t,k,i}$, where i is an index ranging from 0 to $6k$-1. Centers indexed by 0 lie on the horizontal axis staring from S_t towards right, while other centers are indexed counterclockwise (see Fig. 6).

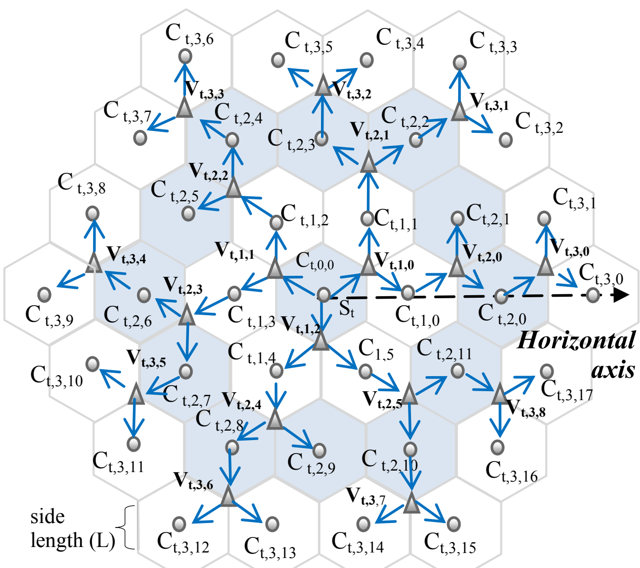


Fig.6 Illustration of 3DOBP in hexagon rings (in layer t)

The relative location $LC_{t,k,i}$ of $C_{t,k,i}$ relative to S_t can be derived handily by a *geometric mapping* $M(C_{t,k,i}) \rightarrow LC_{t,k,i}$. The geometric mapping will be well defined in subsection 3.2. Additionally, the LS_t is $LC_{t,0,0}$ and the LI_t is $(LC_{t-1,0,0}+ LC_{t,0,0})/2$ for t>0 (on the other hand, LI_t is $(LC_{t+1,0,0}+ LC_{t,0,0})/2$ for t<0), where LS_t and LI_t respectively denote the relative locations of starting node S_t and interlayer node I_t, which are relative to the absolute location of the source node S.

In 3DOBP, the source node S_0 (associated with $C_{t,0,0}$) should send the broadcast packet and activate six center nodes (associated with $C_{t,1,0},\ldots,C_{t,1,5}$) in the level-1 ring to forward the packet. And each center node associated with $C_{t,k,i}$ in the level-k ring, $k \geq 1$, should either activate no node or activate two neighboring center nodes in the next level. Actually, for $k \geq 1$, $3(k+1)$ center nodes in the level-k ring need to activate 2 neighboring center nodes in the level-(k+1) ring, while $3(k-1)$ nodes need not to activate any node. For example, all 6 level-1 center nodes need to activate 2 level-2 center nodes, and thus all 12 level-2 center nodes can be activated properly. For another example, 9 (resp., 3) out of 12 level-2 center nodes need to (resp., need not to) activate 2 level-3 center nodes, and thus all 18 level-3 center nodes can be activated properly. We devise a mapping called the *activation target mapping* $T(C_{t,k,i})$ that outputs an empty set or a set $\{C_{t,k+1,w}, C_{t,k+1,w+1}\}$ of two next-level neighboring center nodes for $C_{t,k,i}$, $k \geq 1$, to activate. Note that $C_{t,k+1,w}$ (resp., $C_{t,k+1,w+1}$) must be a neighboring center node of $C_{t,k,i}$; i.e., the associated hexagons of $C_{t,k,i}$ and $C_{t,k+1,w}$ (resp., $C_{t,k+1,w+1}$) must share an edge. The activation target mapping will be well defined in subsection 3.3.

By the node activation process just mentioned, all center nodes can be activated to transmit the packet to cover the entire layer. However, since two center nodes cannot communicate with each other directly, we need intermediate nodes between them for relaying the packet. 3DOBP chooses vertex nodes (i.e., the node nearest to a hexagon vertex) as the intermediate nodes to take the advantage that a vertex node can reach two center nodes (e.g., $V_{t,1,0}$ can reach $C_{t,1,0}$ and $C_{t,1,1}$). In 3DOBP, S takes 3 vertex nodes associated with $V_{t,1,0}$, $V_{t,1,1}$, and $V_{t,1,2}$ as intermediate nodes, while the other center node associated with $C_{t,k,i}$ takes only 1 (or 0) vertex node associated with $V_{k+1,i}$. The relative location $LV_{t,k,i}$ of $V_{t,k,i}$, $k>1$, relative to S can be derived by computing the location of the center of $C_{t,k-1,i}$, $C_{t,k,w}$, and $C_{t,k,w+1}$ if $C_{t,k-1,i}$ should activate $C_{t,k,w}$ and $C_{t,k,w+1}$ (i.e., $T(C_{t,k-1,i})=\{C_{t,k,w}, C_{t,k,w+1}\}$). Note that for k=1, $LV_{t,1,i}$ is the location of the center of S, $C_{t,1,2i}$, and $C_{t,1,2i+1}$ for i=0,1,2.

The broadcast packet of 3DOBP is of the format P(LS, F, FA), where LS is the absolute location of the source node, and F is the set of relative locations of intended forwarding nodes in the next-level ring. And, FA is the set of relative locations of intended interlayer (or starting) nodes in the neighboring layers. Note that each packet is sent along with a unique packet ID so that a node can decide if the packet has ever been received. Also note that the relative locations are delivered along with the indexes of center nodes or vertex nodes. That is, when a location $LC_{t,k,i}$ or $LV_{t,k,i}$ is delivered, the indexes t, k and i are also delivered in the packet. Those indexes are very important for a node to calculate the relative locations of intended forwarding nodes by the activation target mapping and the geometric mapping.

3D Optimized Broadcast Protocol (3DOBP)

The step for the source node S to broadcast a packet P

1. S sends the packet P(LS, F,FA) with F={$LV_{0,1,0}$, $LV_{0,1,1}$, $LV_{0,1,2}$}, FA={LI_1, LI_{-1}}.

Steps for other node X receiving P(LS, F, FA):

1. If X receives P for the first time, it registers P. Otherwise, it drops P and stops.
2. If X is not a node nearest to a location in F or FA, it stops.
3. If X is nearest to a center node associated with $C_{t,k,i}$ of a location in F and $T(C_{t,k,i}) \neq Ø$, then X sends P(LS, F′) and stops, where F′={$LV_{t,k,i}$}.
4. If X is nearest to a vertex node associated with $V_{t,k,i}$ of a location in F, X sends P(LS, F′, Ø) and stops. Indeed, X can calculate $T(C_{t,k-1,i})$= {$C_{t,k,w}$, $C_{t,k,w+1}$} based on indexes k, i and then set F′={$LC_{t,k,w}$, $LC_{t,k,w+1}$}.
5. If X is nearest to an interlayer node associated with I_t of a location in FA, X sends P(LS, Ø, FA′) and stops. X set FA′={LS_{t+1}} if t is positive (i.e. up-direction activation). Otherwise, X set FA′={LS_{t-1}} if t is negative (i.e. down-direction activation).
6. If X is nearest to a starting node associated with S_t of a location in FA, X sends P(LS, F′, FA′) and stops. Indeed, X can calculate and then set F ={ $LV_{t,1,0}$, $LV_{t,1,1}$, $LV_{t,1,2}$}. X set FA′={LI_{t+1}} if t is positive (i.e. up-direction activation). Otherwise, X set FA′={LI_{t-1}} if t is negative.

The article [8] proposes two mechanisms to determine whether a node X is the one nearest to a given location. The first mechanism is to make nodes periodically exchange location information with neighboring nodes so that each node can properly elect the node nearest to the given location. The second mechanism is to enforce a backoff timer which is inversely proportional to the distance between a node's location and the given location. The node nearest to the given location thus has the shortest backoff timer and will earliest issue a response, which in turn prohibits other nodes from responding. When node density is sufficiently high, it is usual that only the node closest to the selected location would retransmit the packet. 3DOBP adopts the second mechanism to determine whether a node is the one nearest to a given location. In this way, a node does not need to exchange location information periodically; a node needs only its own location information for 3DOBP to run properly.

3.2. Geometric Mapping

In this subsection we present the geometric mapping $M(C_{t,k,i})$ that maps a hexagon center node $C_{t,k,i}$ to a location $L_{t,k,i}$ relative to the source node S. Let $Z_{t,0,0}$ denote the location relative to S in the t-layer. As shown in Fig. 7, each hexagon ring can be partitioned into six sectors, indexed by 0,..,5, with each sector having k hexagon centers in the level-k ring. Let $Z_{t,k,q}$ denote the location relative to S of the first hexagon center in the sector q of the level-k hexagon ring in the layer t (e.g., $Z_{1,2,0}$ is the location of the hexagon center on the horizontal line from S towards right in the layer 1). We have $Z_{t,k,q} = (\mathrm{H} \cdot t, k\sqrt{3}\mathrm{L} \cdot \cos(q \cdot 60°), k\sqrt{3}\mathrm{L} \cdot \sin(q \cdot 60°))$ for q=0,..,5, where L is the hexagon side length determined by Eq. (4), and H is the height of hexagon prism determined by Eq. (5). Since each sector has k hexagon centers, we can figure out that hexagon center $C_{t,k,i}$ is within sector q, where $q = \lfloor i/k \rfloor$. Now we can define the geometric mapping $M(C_{t,k,i})$ as follows. (Note that "+" represents the vector addition operator in the mapping and the following location calculations.)

Let $q = \lfloor i/k \rfloor$. If i is a mutiple of k, $\mathrm{M}(\mathrm{C}_{t,k,i}) = \mathrm{Z}_{t,k,q}$. Otherwise, $\mathrm{M}(\mathrm{C}_{t,k,i})$

$$= \begin{cases} Z_{t,k,0} + (0, -i\frac{\sqrt{3}\mathrm{L}}{2}, i\frac{3\mathrm{L}}{2}), \text{if } q = 0 \\ Z_{t,k,1} + (0, (k-i)\sqrt{3}\mathrm{L}, 0), \text{if } q = 1 \\ Z_{t,k,2} + (0, (q \cdot k - i)\frac{\sqrt{3}\mathrm{L}}{2}, (q \cdot k - i)\frac{3\mathrm{L}}{2}), \text{if } q = 2 \\ Z_{t,k,3} + (0, (i - q \cdot k)\frac{\sqrt{3}\mathrm{L}}{2}, (q \cdot k - i)\frac{3\mathrm{L}}{2}), \text{if } q = 3 \\ Z_{t,k,4} + (0, (i - q \cdot k)\sqrt{3}\mathrm{L}, 0), \text{if } q = 4 \\ Z_{t,k,5} + \left(0, (i - q \cdot k)\frac{\sqrt{3}\mathrm{L}}{2}, (i - q \cdot k)\frac{3\mathrm{L}}{2}\right), \text{if } q = 5 \end{cases}$$

In Fig. 7, we illustrate the above mapping by two examples. The first example is about $M(C_{0,2,1})$. Since $q = \lfloor 1/2 \rfloor = 0$, we calculate $\mathrm{Z}_{0,2,0} = \left(0, 2\sqrt{3}\mathrm{L} \cdot \cos(0), 2\sqrt{3}\mathrm{L} \cdot \sin(0)\right) = (0, 2\sqrt{3}\mathrm{L}, 0)$. We then have $\mathrm{M}(\mathrm{C}_{0,2,1}) = \mathrm{Z}_{0,2,0} + \left(0, -\frac{\sqrt{3}\mathrm{L}}{2}, \frac{3\mathrm{L}}{2}\right)$. The second example is about $M(C_{0,2,7})$. Since $q = \lfloor 7/2 \rfloor = 3$, we calculate $\mathrm{Z}_{0,2,3} = \left(0, 2\sqrt{3}\mathrm{L} \cdot \cos(180°), 2\sqrt{3}\mathrm{L} \cdot \sin 180° = (0, -23\mathrm{L}, 0)\right)$. We then have $\mathrm{M}(\mathrm{C}_{0,2,7}) = \mathrm{Z}_{0,2,3} + \left(0, \frac{\sqrt{3}\mathrm{L}}{2}, -\frac{3\mathrm{L}}{2}\right)$.

3.3. Activation Target Mapping

In this subsection we present the activation target mapping $T(C_{t,k,i})$. The input of $T(C_{t,k,i})$ is a center node $C_{t,k,i}$ for $k \geq 1$. $T(C_{t,k,i})$ is to find two next-level neighboring center nodes $C_{t,k+1,w}$ and $C_{t,k+1,w+1}$ of $C_{t,k,i}$, where w is even. If such neighboring nodes exit, the output of $T(C_{t,k,i})$ is { $C_{t,k+1,w}$, $C_{t,k+1,w+1}$}; otherwise, the output is an empty set. For example, $T(C_{0,1,0})$={$C_{0,2,0}$, $C_{0,2,1}$} since $C_{t,k+1,w}$ is $C_{0,2,0}$ and $C_{t,k+1,w+1}$ is $C_{0,2,1}$ with w=0. For another example, $T(C_{0,2,1})$=Ø, since the index w of the neighboring center nodes ($C_{0,3,1}$ and $C_{0,3,2}$) of $C_{0,2,1}$ is odd.

As shown in Fig. 7, each hexagon ring can be partitioned into six sectors, indexed by 0,..,5, each having a starting axis (i.e., A_0, ,…, A_5). Let $q = \lfloor i/k \rfloor$ denote the index of the sector in which $C_{t,k,i}$ resides. $T(C_{t,k,i})$ is defined as follows.

$T(C_{t,k,i})=$

$$\begin{cases} \{C_{t,k+1,i}, C_{t,k+1,i+1}\}, & \text{if } q=0 \text{ and } i \text{ is even} \\ \{C_{t,k+1,i+1}, C_{t,k+1,i+2}\}, & \text{if } q=1 \text{ and } i \text{ is odd} \\ \{C_{t,k+1,i}, C_{t,k+1,i+1}\}, & \text{if } q=1, (i \bmod k)=0 \text{ and } i \text{ is even} \\ \{C_{t,k+1,i+2}, C_{t,k+1,i+3}\}, & \text{if } q=2 \text{ and } i \text{ is even} \\ \{C_{t,k+1,i+3}, C_{t,k+1,i+4}\}, & \text{if } q=3 \text{ and } i \text{ is odd} \\ \{C_{t,k+1,i+2}, C_{t,k+1,i+3}\}, & \text{if } q=3, (i \bmod k)=0 \text{ and } i \text{ is even} \\ \{C_{t,k+1,i+4}, C_{t,k+1,i+5}\}, & \text{if } q=4 \text{ and } i \text{ is even} \\ \{C_{t,k+1,i+5}, C_{t,k+1,i+6}\}, & \text{if } q=5 \text{ and } i \text{ is odd} \\ \{C_{t,k+1,i+4}, C_{t,k+1,i+5}\}, & \text{if } q=5, (i \bmod k)=0 \text{ and } i \text{ is even} \\ \emptyset, & \text{otherwise} \end{cases}$$

The definition of $T(C_{t,k,i})$ contains 10 cases of different conditions. The first case (q=0 and i is even) drives the center node $C_{t,k,i}$ to activate two neighboring center nodes $C_{t,k+1,i}$ and $C_{t,k+1,i+1}$ in the next level, only if $C_{t,k,i}$ is in sector 0 and i is even. The second case (q=1 and i is odd) drives $C_{t,k,i}$ to activate $C_{t,k+1,i+1}$ and $C_{t,k+1,i+2}$, only if $C_{t,k,i}$ is in sector 1 and i is odd. The third case (q=1, (i mod k)=0 and i is even) drives $C_{t,k,i}$ to activate $C_{t,k+1,i}$ and $C_{t,k+1,i+1}$, only if $C_{t,k,i}$ is in sector 1, $C_{t,k,i}$ is on the starting axis of sector 1 (i.e. A1), and i is even. Similarly, the fourth, ..., and the ninth cases drive $C_{t,k,i}$ in the sectors 2, …, and 5 to activate two center nodes for specific conditions. The last case drives $C_{t,k,i}$ not to active any node, only if none of the first nine conditions is satisfied.

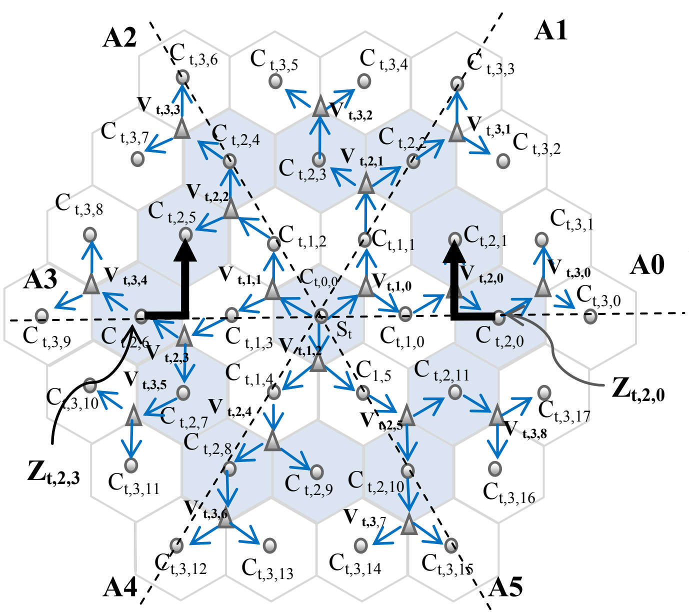


Fig. 7 The illustration of the Geometric Mapping and Activation Target Mapping

In Fig. 7, we illustrate $T(C_{t,k,i})$ by three examples. The first example is about $T(C_{0,2,0})$. Let $q=\lfloor 0/2 \rfloor=0$. Since q is 0 and i (=0) is even, we have $T(C_{0,2,0})=\{C_{0,3,0}, C_{0,3,1}\}$. The second example is about $T(C_{0,2,5})$. Let $q=\lfloor 5/2 \rfloor=2$. Since q is 2 and i (=5) is not even, we have $T(C_{0,2,5})=\emptyset$, which means the node $C_{0,2,5}$ needs not to activate any node. The third example is about $T(C_{0,2,2})$. Let $q=\lfloor 2/2 \rfloor=1$. Since q is 1, i (=2) is even, and i is a multiple of k (=2), we have $T(C_{0,2,2})=\{C_{0,3,2}, C_{0,3,3}\}$.

4. Performance Evaluation

In this section, we first derive the theoretical upper bound of transmission efficiency for 3D wireless networks. We then analyze and compare the transmission efficiency of the proposed 3DOBP and the 3D broadcast protocols based on filling space by polyhedrons like cubes, hexagonal prisms, rhombic dodecahedrons, and truncated octahedrons.

4.1. Transmission efficiency upper bound

The transmission efficiency is defined as the effective transmission region over the sum of the transmission regions. We use two spheres A and B of radius R in Fig. 8 to derive the transmission efficiency upper bound.

According to [13], the intersected volume of two spheres is as follows:

$$V_{\text{inter}}(R, d) = \frac{1}{12}\pi(4R+d)(2R-d)^2 \tag{6}$$

Where R is the radius of sphere and d is the distance of two spheres.

To minimize the intersected volume, while keeping the two sphere center within distance R is d=R. Hence, the upper bound of transmission efficiency (TE) in the 3D space is as follows:

$$TE(R) = \frac{2V_{\text{sphere}}(R) - V_{\text{inter}}(R,R)}{2V_{\text{sphere}}(R)} \tag{7}$$

Where V_{sphere} is the volume of sphere with radius R, which is $(4/3)\pi R^3$.

Letting R=1, we have the upper bound of transmission efficiency in 3D as follows:

$$TE = 1 - \frac{(5/12)\pi}{2(4/3)\pi} = \frac{27}{32} = 0.84375 \tag{8}$$

Thus, we have that the transmission efficiency upper bound is 0.84375. It means each transmission can reach 84% extra region.

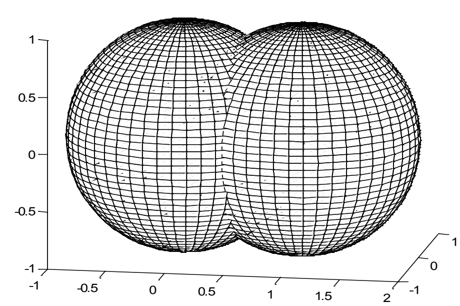


Fig.8 Intersected Sphere, where center of A is at (0,0,0) and B is at (1,0,0), the distance between two centers is one unit distance.

4.2. Transmission efficiency of cubes

We assume a cube is with the side length L. The center of each cube is located by a node of transmission radius R. We can form a circumsphere for a cube, where the radius of circumsphere is Rc. The relationship between the side length and the radius of circumsphere is: $\mathrm{L} = 2\mathrm{Rc}/\sqrt{3}$. The distance between two neighboring cube centers should be within R. Thus, $\mathrm{R} = \mathrm{L} = 2\mathrm{Rc}/\sqrt{3}$.

For a set of *N* nodes with the cube arrangement, the transmission efficiency is:

$$\frac{N\cdot \mathrm{L}^3}{N\cdot\pi(2\mathrm{Rc}/\sqrt{3})^3\cdot 4/3} = \frac{8\mathrm{Rc}^3/3\sqrt{3}}{\pi 8\mathrm{Rc}^3/3\sqrt{3}\cdot 4/3} = \frac{3}{4\pi} = 0.238732 \qquad (9)$$

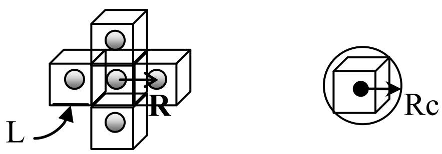


Fig. 9 Illustration of the cube arrangement, where L is the side length, R is the transmission radius, and Rc is the radius of the circumsphere of the cube cell

4.3 Transmission efficiency of hexagonal prisms

We assume a hexagonal prism is with side length L and height H, and that the center of each hexagonal prism is located by a node with transmission radius R. By Eq. (2), the hexagon prism has the maximal volume when $\mathrm{H} = \mathrm{L}\sqrt{2}$. We have the volume of hexagonal prism (HPV) is $\frac{3\sqrt{3}}{2}\mathrm{L}^2\cdot\mathrm{L}\sqrt{2} = \frac{3\sqrt{3}}{\sqrt{2}}\mathrm{L}^3$. Moreover, we set the radius Rc of the circumsphere of the hexagonal prism as follows:

$$\mathrm{Rc} = \sqrt{\mathrm{L}^2 + \mathrm{H}^2/4} = \sqrt{\mathrm{L}^2 + \mathrm{L}^2/2} = \sqrt{3\mathrm{L}^2/2} = \mathrm{L}\sqrt{3/2} \qquad (10)$$

And, the relationship between the transmission radius and the circumsphere radius is $\mathrm{R} = \sqrt{2}\mathrm{Rc}$. We have the following equation:

$$\mathrm{R} = \sqrt{2}\mathrm{Rc} = \sqrt{2}\mathrm{L}\sqrt{\frac{3}{2}} = \sqrt{3}\mathrm{L} \qquad (11)$$

For a set of *N* nodes with hexagonal prism arrangement, the transmission efficiency is as follows:

$$\frac{N\cdot\mathrm{HPV}}{N\cdot\pi\mathrm{R}^3\cdot 4/3} = \frac{\frac{3\sqrt{3}}{\sqrt{2}}\mathrm{L}^3}{\pi\cdot(\sqrt{3}\mathrm{L})^3\cdot 4/3} = \frac{\frac{3\sqrt{3}}{\sqrt{2}}}{\pi\cdot(\sqrt{3})^3\cdot 4/3} = \frac{3}{4\sqrt{2}\pi} = 0.168809 \qquad (12)$$

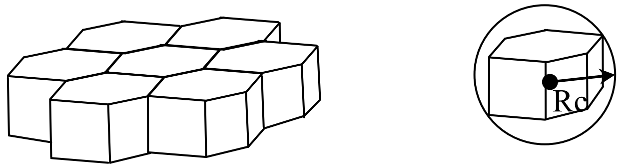


Fig. 10 Illustration of the hexagonal prism arrangement

4.4 Transmission efficiency of rhombic dodecahedrons

A rhombic dodecahedron can be constructed by two cubes of the length L. Based on the results of [1], we have the following settings. We can construct a circumsphere for a rhombic dodecahedron. The radius Rc of the circumsphere is L. The volume RDV of a rhombic dodecahedron is $2\mathrm{L}^3$. The transmission radius R of a rhombic dodecahedron center node is $\sqrt{2}$Rc. Since Rc=L, we have $\mathrm{R}=\sqrt{2}\mathrm{L}$.

For a set of *N* nodes with rhombic dodecahedron arrangement, the transmission efficiency is:

$$\frac{N\cdot\mathrm{RDV}}{N\cdot\pi\mathrm{R}^3\cdot 4/3} = \frac{2\mathrm{L}^3}{\pi\cdot(\sqrt{2}\mathrm{L})^3\cdot 4/3} = \frac{1}{\pi\cdot\sqrt{2}\cdot 4/3} = \frac{3}{4\sqrt{2}\pi} = 0.168809 \qquad (13)$$

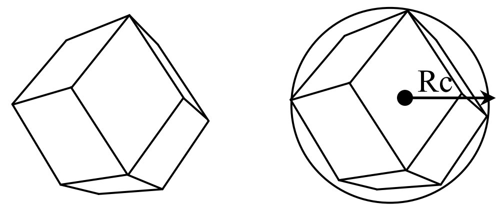


Fig. 11 Illustration of a rhombic dodecahedron cell

4.5 Transmission efficiency of truncated octahedrons

We assume that a truncated octahedron is of side length L and the center of each truncated octahedron is located by a node whose transmission radius is R. Let Rc be the radius of the circumsphere of a truncated octahedron. Based on the results of [1], we set $\mathrm{Rc} = \mathrm{L}\sqrt{10}/2$ and $\mathrm{R} = 4\mathrm{Rc}/\sqrt{5}$. Then we have $\mathrm{R} = \frac{4}{\sqrt{5}}\cdot\frac{\mathrm{L}\sqrt{10}}{2} = 2\mathrm{L}\sqrt{2}$.

For a set of *N* nodes with hexagon prism arrangement, the transmission efficiency is:

$$\frac{N\cdot 8\sqrt{2}\mathrm{L}^3}{N\cdot\pi(2\mathrm{L}\sqrt{2})^3\cdot 4/3} = \frac{N\cdot 8\sqrt{2}}{N\cdot\pi(2\sqrt{2})^3\cdot 4/3} = \frac{3}{8\pi} = 0.119366 \qquad (14)$$

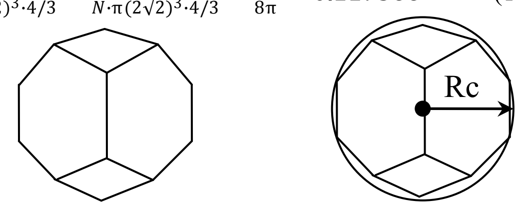


Fig. 12 Illustration of a truncated octahedron cell

4.6 Transmission efficiency of 3DOBP

We assume a hexagonal prism is with side length L and height H, and that the center of each hexagonal prism is located by a node with transmission radius R. By Eqs. (4) and (5), we have $\mathrm{L} = \frac{\mathrm{R}}{\sqrt{3/2}}$ and $\mathrm{H} = 2\frac{\mathrm{R}}{\sqrt{3}}$. The volume HPV of a hexagonal prism is:

$$\mathrm{HPV} = \frac{3\sqrt{3}}{2}\mathrm{L}^2\cdot\mathrm{H} = \frac{3\sqrt{3}}{2}\mathrm{L}^2\cdot\mathrm{L}\sqrt{2} = \frac{3\sqrt{3}}{\sqrt{2}}\mathrm{L}^3 \qquad (15)$$

With the help of hexagon rings, in 3DOBP, all center nodes suffice to cover a layer of the space. We assume the number of center nodes is N. Then we need a number of N/2 of vertex nodes to be activated to forward packets to the N center nodes. Let M be the number of center nodes and activated vertex nodes. We have

$$\mathrm{N} = \sum_{k=1}^{\mathrm{H}} 6k = 3\mathrm{H}(\mathrm{H}+1) \qquad (16)$$

$$M = \sum_{k=1}^{H} 6k + \frac{1}{2}\sum_{k=1}^{H} 6k = \frac{3}{2}3H(H+1) \tag{17}$$

To simplify the calculation, we assume the number of interlayer nodes is ignorable. The transmission efficiency of the proposed protocol is

$$TR = \frac{N \cdot HPV}{M \cdot \pi R^3 \cdot \frac{4}{3}} = \frac{(3H(H-1)) \cdot HPV}{(\frac{3}{2}3H(H-1)) \cdot \pi R^3 \cdot 4/3} = \frac{2 \cdot HPV}{3 \cdot \pi R^3 \cdot 4/3} = \frac{\frac{3\sqrt{3}}{\sqrt{2}}L^3}{2\pi(L\sqrt{3/2})^3} = \frac{1}{\pi} = 0.31831 \tag{18}$$

4.7 Transmission efficiency comparison

In this subsection, we compare 3DOBP with other 3D broadcast protocols based on filling space by polyhedrons like cubes [1], hexagonal prisms [1, 2], rhombic dodecahedrons [1, 3], and truncated octahedrons [1] in terms of transmission efficiency. Note that the comparisons do not include the 3D broadcast protocol proposed in [4, 9], since the aggregate transmission space of retransmitting nodes does not fully cover the network space, for the protocol does not consider the optimal setting of the height of hexagonal prisms. The comparison results are summarized in the Table 1. We can observe that 3DOBP is with the highest transmission efficiency.

Table 1. Comparisons of transmission efficiency

Approach	Transmission efficiency
Cube based	$3/4\pi \approx 0.238732$
Hexagonal Prism based	$3/(4\sqrt{2}\pi) \approx 0.168809$
Rhombic Dodecahedron based	$3/(4\sqrt{2}\pi) \approx 0.168809$
Truncated Octahedron based	$3/8\pi \approx 0.119366$
3DOBP	$1/\pi \approx 0.31831$

5. Conclusion

Broadcasting information throughout the network is a fundamental operation of the wireless network. In this paper, we have first generalized the definition of transmission efficiency for 3D wireless networks and shown that the theoretical upper bound of transmission efficiency is 0.84. We have further proposed an optimized broadcasting protocol, called 3D optimized broadcast protocol (3DOBP), by partitioning the 3D space into multi-layer hexagonal prisms of a hexagon ring pattern in each layer. As we have shown, the transmission efficiency of the proposed protocol can reach $1/\pi$, which is better than those of other polyhedron-filling approaches using cubes, hexagon prisms, rhombic dodecahedrons, and truncated octahedrons.

References

[1] S. M. N. Alam and J. H. Zygmunt, "Coverage and connectivity in three-dimensional networks," *Mobicom 2006*. Los Angeles, CA, 2006.

[2] C. Decayeux and D. Seme, "A new model for 3D cellular mobile networks," *Workshop on Parallel and Distributed Computing, Third International Symposium on/Algorithms, Models and Tools for Parallel Computing on Heterogeneous Networks*, 2004.

[3] J. Carle, J.F. Myoupo, and D. Semé, "A Basis for 3-D Cellular Networks," in *Proc. of the 15th International Conference on Information Networking*, 2001.

[4] A. Durresi, V. Paruchuri, L. Barolli, and R. Jain, "Air to air communication protocol," *IEEE Aerospace Conference*, 2006.

[5] Durresi, V. Paruchuri, R. Kannan, S.S. Iyengar, "Optimized Broadcast Protocol for Sensor Networks," *IEEE Transactions on Computers*, Volume 54, Issue 8, pp. 1013 – 1024, Aug. 2005.

[6] S.-Y. Ni, Y.-C. Tseng, Y.-S. Chen, and J.-P. Sheu, "The broadcast storm problem in a mobile ad hoc network," in *Proc. of the 5th Annual ACM/IEEE International Conference on Mobile Computing and Networking*, 1999.

[7] D. Kim and N. F. Maxemchuk, "A comparison of flooding and random routing in mobile ad hoc network," in *Proc. of Third New York Metro Area Networking Workshop*, 2003.

[8] V. Paruchuri, "Adaptive scalable protocols for heterogeneous wireless networks," *PhD dissertation, Louisiana State University*, 2006.

[9] V. Paruchuri, A. Durresi, L. Barolli, and M. Takizawa, "Three Dimensional Broadcast Protocol for Wireless Networks," in *Proc. of International Conference on Parallel Processing*, 2007.

[10] V. Paruchuri, A. Durresi, D. S. Dash, and R. Jain, "Optimal flooding protocol for routing in ad-hoc networks," in *Proc. of IEEE Wireless Communications and Networking Conference*, 2003.

[11] M. K. Watfa and S. Commuri, "Optimal 3-dimensional sensor deployment strategy," in *Proc. of IEEE Consumer Communications and Networking Conference*, 2006.

[12] Yung-Liang Lai and Jehn-Ruey Jiang, "Broadcasting with Optimized Transmission Efficiency in Wireless Networks," to be presented in *Fifth International Conference on Wireless and Mobile Communications*, 2009.

[13] Eric W. Weisstein, "Sphere-Sphere Intersection," from MathWorld -- A Wolfram Web Resource. http://mathworld.wolfram.com/Sphere-SphereIntersection.html