A 3-Dimensional Broadcast Protocol with Optimized Transmission Efficiency in Wireless Networks

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Abstract: Broadcasting is one of the most important operations in the wireless network for disseminating information throughout the entire network. Flooding is a simple mechanism to realize broadcasting, but it has high redundancy of retransmissions, leading to low transmission efficiency. Many broadcast protocols have been proposed for pursuing optimized transmission efficiency for wireless networks hypothetically deployed on the 2-dimensional (2D) plane. In some applications (e.g., a multi-storey building), however, wireless networks are deployed in the 3D space. In this paper, we derive the upper bound of 3D transmission efficiency and propose a 3D broadcast protocol, called 3DOBP (3-Dimensional Optimized Broadcast Protocol), to achieve optimized transmission efficiency by partitioning the 3D space into multi-layer hexagonal prisms of a hexagon ring pattern in each layer. As we will show, the transmission efficiency of the proposed protocol is $1/\pi$, which is better than those of polyhedron-filling 3D broadcasting approaches using cubes, hexagon prisms, rhombic dodecahedrons, and truncated octahedrons. To the best our knowledge, the proposed broadcast protocol is the one with the highest 3D transmission efficiency so far.

Keywords: Broadcasting; transmission efficiency; wireless networks; optimization

1 Introduction

Broadcasting is one of the most important operations for disseminating data or control signals among nodes of a wireless network. Flooding is an intuitive approach to realize broadcasting, in which each node retransmits a packet when receiving it for the first time. Flooding is simple and is highly reliable; however, it may cause the broadcast storm problem (Ni et al., 1999) and has low *transmission efficiency* due to redundancy of retransmissions. As shown in (Kim and Maxemchuk, 2003), the theoretical upper bound of transmission efficiency is 0.61 for wireless networks deployed on the 2-dimensional (2D) plane.

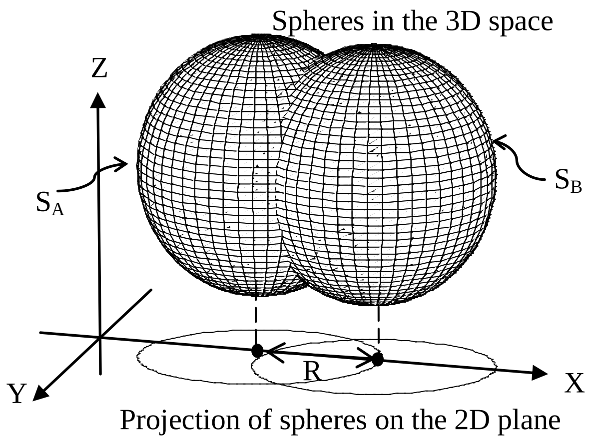
Some geometry-based broadcast protocols (Durresi et al., 2005; Kim and Maxemchuk, 2003; Lai and Jiang, 2009; Paruchuri et al., 2003) for 2D wireless networks have been proposed to pursue optimized transmission efficiency. Among them, the Optimized Broadcast Protocol (OBP) proposed in (Lai and Jiang, 2009) achieves the highest transmission efficiency 0.55, which is about 90% of the theoretical upper bound. In OBP, a node relies on hexagon rings centered at the broadcasting source node to decide if it should retransmit a broadcast packet when it receives the packet for the first time.

In some environments, the wireless network is deployed in the 3-dimensional (3D) space instead of the 2D plane. A wireless network consisting of devices distributed in different floors of a multi-storey factory is a typical example. In such a 3D wireless network, the relationship between wireless communicating nodes should be 3D rather than 2D. This motivates us to investigate broadcasting and the transmission efficiency for 3D wireless networks.

Some papers (Alam and Hass, 2006; Carle et al., 2001, Decayeux and Seme, 2004; Hatzis et al., 1999; Watfa and Commuri, 2006) study the arrangement of nodes to fully cover a given space with optimized number of nodes. Most of them (Alam and Hass, 2006; Carle et al., 2001, Decayeux and Seme, 2004; Hatzis et al., 1999) are based on the regular polyhedron filling approach, which partitions the space into 3D cells, each being a polyhedron located by a node in the center. The paper (Alam and Hass, 2006) investigated how to fill the space with regular polyhedrons, such as *cubes*, *hexagonal prisms*, *rhombic dodecahedrons*, and *truncated octahedrons*. The cube is used as a fundamental element to model a 3D wireless network in (Hatzis et al., 1999). The hexagonal prism, rhombic dodecahedron are studied in (Carle et al., 2001) and (Decayeux and Seme, 2004) for planning base stations to fully cover a 3D wireless network. The paper (Watfa and Commuri, 2006) proposes using the *body-center cubic (BCC)* structure to fill the space; interestingly, filling space by BCCs is equivalent to filling space by truncated octahedrons.

When the center nodes of neighboring cells can communicate with each other, the polyhedron filling approach can be transformed into a 3D broadcast protocol by demanding the center node in each cell to retransmitting the broadcast packet. We can thus design 3D broadcast protocols with the help of the above-mentioned regular polyhedrons to achieve good transmission efficiency. Such protocols have 100% *coverage ratio*, the ratio of the network space that is covered by the transmission ranges of the source node or the retransmitting nodes, when the node density is sufficiently high. The paper (Durresi et al., 2006) adopts a different strategy to designing a 3D broadcast protocol, called *Air to Air Communication Protocol (AACP)*, which requires vertex nodes, instead of center nodes, in hexagonal prisms to retransmit the broadcast packet for better transmission efficiency. Durresi et al. also presented the best condition to maximize the reachability. Note that a similar protocol is also proposed in (Paruchuri et al., 2007).

Figure 1 Illustration of the upper bound case of transmission efficiency in the 3D space

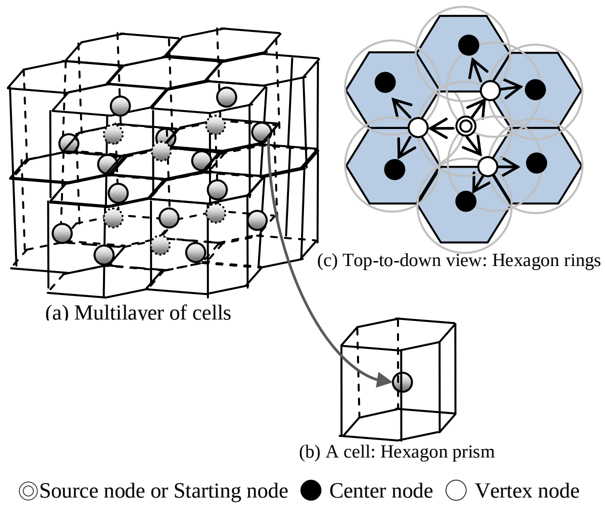


In this paper, we generalize the definition of transmission efficiency for 3D wireless networks. To take two communicating nodes A and B in Fig. 1 as an example, the transmission efficiency is the ratio of the effective communication area (the region covered by spheres S_A or S_B, i.e., $|S_A \cup S_B|$) over the total communication region (the summation of volumes of S_A and S_B, i.e., $|S_A|+|S_B|$), where S_A and S_B are the spheres centered respectively at A and B with the radius of R, the transmission range. When the distance between nodes A and B equals to the transmission range R, transmission efficiency reaches the upper bound. As we will show, the theoretical upper bound is 0.84 in the 3D space.

We further propose an optimized broadcast protocol, called *3D optimized broadcast protocol (3DOBP)*, by partitioning the 3D space into multi-layer hexagonal prisms. As shown in Fig. 2(a), the network space is partitioned into multiple layers along the vertical axis (i.e., Z axis). And each layer consists of a set of cells each of which is a hexagonal prism, as shown in Fig. 2(b). From the top-to-down view, the cells in a layer form hexagon rings. As shown in Fig. 2(c), the hexagon rings of the middle layer is centered at the *source node* S (represented as ◎) that initiates the broadcast of a packet. From the source node and along the vertical axis, there is a *starting node* nearest to the center of a cell for each upper or lower layer. Only the starting nodes, the *vertex nodes* nearest to hexagon centers (represented as ●) and specific vertex nodes nearest to some hexagon vertexes (represented as ◯) need to retransmit the packet. We carefully analyze the transmission efficiency of our proposed protocol and other polyhedron-filling approaches using cubes, hexagon prisms, rhombic dodecahedrons, and truncated octahedrons. As we will show, the transmission efficiency of the proposed protocol can reach $1/\pi$. Compared with the other polyhedron-filling approaches, our proposed protocol is with the highest transmission efficiency.

The rest of this paper is organized as follows. In Section 2, we introduce some related work. In Section 3, we present the proposed protocol. In Section 4, we analyze transmission efficiency of the proposed protocol and compare the analyzed result with those of polyhedron-filling based approaches. We demonstrate simulation results in Section 5. And finally, some concluding remarks are drawn in Section 6.

Figure 2 Illustration of partitioning the 3D space into multi-layer hexagon prisms of the hexagon ring pattern



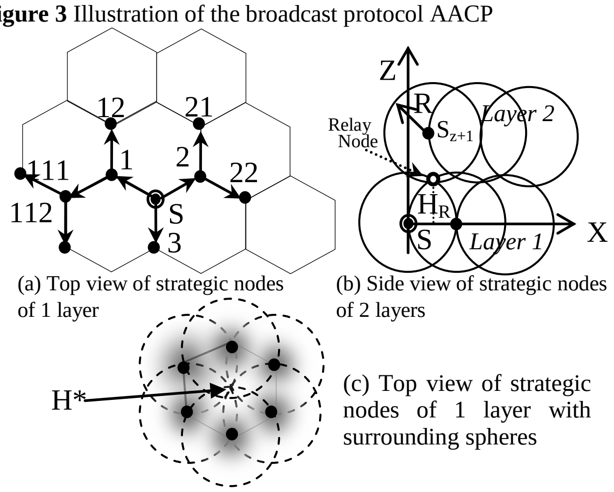
2 RELATED WORK

The paper (Hatzis et al., 1999) uses a cube as a fundament block to model a three dimensional network. It uses a cube as the approximation of a sphere of a node's transmission space on the basis of the fact that a 3D space can be quantized by a set of cubes in geometry. The papers (Carle et al., 2001) and (Decayeux and Seme, 2004) study the base station arrangement problem for 3D picocellular networks. They rely on the hexagonal prism pattern and the rhombic dodecahedron pattern, respectively, for positioning base stations. The paper (Alam and Hass, 2006) studies the sensor deployment problem for 3D wireless sensor networks. The problem is to select the minimum number of sensors for covering a given space, while keeping sensors connected. The solution can be solved on the basis of Kelvin's conjecture. In 1887, Kelvin raised a question: "What is the optimal way to fill a three dimensional space with cells of equal volume, so that the surface area (interface area) is minimized?" Kelvin merely conjectured (but did not proved) that using truncated octahedrons to fill the space is optimal. In (Alam and Hass, 2006), truncated octahedrons are used for 3D sensor network deployment. Moreover, cubes, hexagonal prisms and rhombic dodecahedrons are also applied to the deployment for the sake of comparison. The structures are analyzed in terms of the number of cells needed to cover a given space. By the analysis, the truncated octahedron indeed presents the minimum number of nodes for deployment. The paper (Watfa and Commuri, 2006) proposes using the body-center cubic (BCC) structure for 3D sensor network deployment. By BCC, sensors are deployed at eight vertexes of a cube and at the center of the cube. As shown in (Alam and Hass, 2006), the BCC-based deployment is equivalent to truncated octahedron-based deployment by the Voronoi tessellation analysis.

The paper (Durresi et.,2006) proposes a 3D broadcast protocol, called *air to air communication protocol (AACP)*, which partitions the space into multiple layers (a layer is a subspace partitioned by the same height of a space), each of which is covered by a set of selected nodes. The selected nodes lie on a same plane and their locations are decided by the regular hexagon lattices, as shown in Fig. 3(a). Only the nodes at (or nearest to) hexagon vertexes need to retransmit the broadcast packet. To take the scenario in Fig. 3(a) for an example, a source node (S) initially issues the broadcast packet destined to locations (1), (2), and (3). The node nearest to the location (1) is responsible for retransmitting the packet. Then, the node nearest to the location (11) is responsible for the retransmission. The nodes nearest to the locations (111) and (112) are then responsible for the retransmission. Similarly, the other nodes nearest to the locations of hexagon vertexes are responsible for the retransmission and hence one layer of a plane is fully covered.

As shown in Fig. 3(b), the entire space is divided into layers (subspace) of height H_R. Each layer is covered by the procedures illustrated in Fig. 3(a). The source node S also triggers a forward node on the upper (or lower) sub-region. In Fig. 3(b), the node S has to trigger node S_{z+1}. When the distance between S and S_{z+1} is more than transmission range R, a relay node is required to be activated. Note that the paper (Paruchuri et al., 2007) also reports a similar protocol.

Figure 3 Illustration of the broadcast protocol AACP



(a) Top view of strategic nodes of 1 layer

(b) Side view of strategic nodes of 2 layers

(c) Top view of strategic nodes of 1 layer with surrounding spheres

The AACP protocol is transmission-efficient, as it partitions the space into multiple layers and tries to minimize the number of retransmissions for covering a layer by selecting a small number of *strategic nodes* to retransmit the broadcast packet. The selection of strategic nodes is based on the classical *two-dimensional circle covering problem* in (Kershner, 1939), which asks "How to arrange circles such that the minimum number of circles can completely cover a given area?" The paper (Kershner, 1939) showed that the regular hexagon lattice based arrangement is the best solution to the two-dimensional circle covering problem. AACP bases on such a hexagon lattice to select the nodes located at the hexagon vertices as the strategic nodes to keep as small as possible the number of strategic nodes and to keep the nodes' connectivity. The paper (Durresi et al., 2006) also presented the best performance achieved in AACP to reach a very high reachability in broadcasting is at $H_R = 0.8 \times R$. At this value, AACP achieved very high reachability and a further reduction of H_R does not increase considerably the reachability. The reason is the fact that the central upper/lower region of each hexagon (i.e., the region above/below the location H* shown in Fig.3(c)) is hard to be covered, for nodes are deployed at the hexagon vertexes, which are far from the hexagon center. Our technical report (Lai & Jiang, 2011) also has reported the same result.

We can certainly reduce the distance between two layers to eliminate the uncovered space problem. However, the number of overall strategic nodes is thus increased, and the transmission efficiency is in turn reduced. It remains open for AACP to decide the optimal value of the distance between two layer planes such that the network space is fully covered and the transmission efficiency is maximized.

3 The Proposed Protocol

In this section, we describe the proposed protocol, 3-Dimensional Optimized Broadcast Protocol (3DOBP). We assume that each node is aware of its location but is not aware of the network topology. We also assume that all nodes are equipped with omnidirectional antennas, whose transmission ranges are of the same value R and are modeled as spheres of the radius R.

3.1 Overview of 3DOBP

In 3DOBP, the entire network space is divided into multiple layers of height H. Each layer consists of a set of cells, as shown in Fig. 4(a). As shown in Fig. 4(b), each cell is a hexagonal prism of the side length L and the height H. Fig. 4(c) shows that the *source node* or the *initial starting node* (represented as ◎) is in the middle layer (layer 0) to broadcast a packet and the *interlayer nodes* (represented as ⊗) is between layers to forward the packet to other *starting nodes* (also represented as ◎) in different layers (layers 1, -1, ⋯, etc.) At a specific layer, only the nodes nearest to hexagon centers (represented as ●) and the nodes nearest to some specific hexagon vertexes (represented as ○) need to retransmit the packet. Note that below we use the term *center node* (resp., *vertex node*) to stand for the node nearest to a hexagon center (resp., vertex).

We assume there is a node with transmission radius R located at the center of a hexagonal prism of side length L and height H. We can draw a circumsphere of radius Rc for the hexagonal prism. The relationship of L, H and Rc is: $\mathrm{Rc} = \sqrt{L^2 + H^2/4}$. We set V-Ratio to represent ratio of the volume of the hexagonal prism over the volume of the circumsphere as follows.

$$\text{V-Ratio} = \frac{3\sqrt{3}}{2}L^2H/(\frac{4\pi}{3}\left(\sqrt{L^2+\frac{H^2}{4}}\right)^3) \tag{1}$$

When V-Ratio is maximized, the volume of the hexagonal prism is maximized. So, we have to derive the first derivative of V-Ratio equation, and then find its solution. After some basic calculations, we have the solution

$$H=\sqrt{2}L \tag{2}$$

We set the transmission radius R to be the circumsphere radius Rc. We then have

$$R=Rc=L\sqrt{3/2} \tag{3}$$

$$L=R/\sqrt{3/2} \tag{4}$$

$$H=L\sqrt{2}=\frac{R}{\sqrt{3/2}}\sqrt{2} = 2\frac{R}{\sqrt{3}} \tag{5}$$

To sum up, we can use a hexagonal prism of side length $R/\sqrt{3/2}$ and height $2\ R/\sqrt{3}$ to represent the *effective transmission space* of a node of transmission radius R.

To broadcast a packet throughout the entire network space, we need to achieve the following two goals: (a) to activate starting nodes for retransmission in every layer along the Z-axis, and (b) to activate center nodes in every layer for retransmission. Below, we show how to achieve the two goals.

How to activate staring nodes in different layers

We partition the network space into multiple layers. The source node or the initial starting node S_0, which initiates the broadcast of a packet, is assumed to be in the layer 0. S_0 is required to activate the starting node S_t in every layer t with the help of interlayer nodes. S_0 just transmits the packet to its two neighboring interlayer nodes I_1 and I_{-1} in layer 1 and layer -1, respectively. Then, I_1 will forward the packet to starting node S_1 in layer 1 to activate it. Similarly, I_{-1} will forward the packet to the starting node S_{-1} in layer -1 to activate it. Likewise, stating nodes S_1 (or S_{-1}) will active starting node S_2 (or S_{-2}), and so on. So, the starting node in every layer will be activated along the Z-axis.

Figure 4 Illustration of space partitioning of 3DOBP

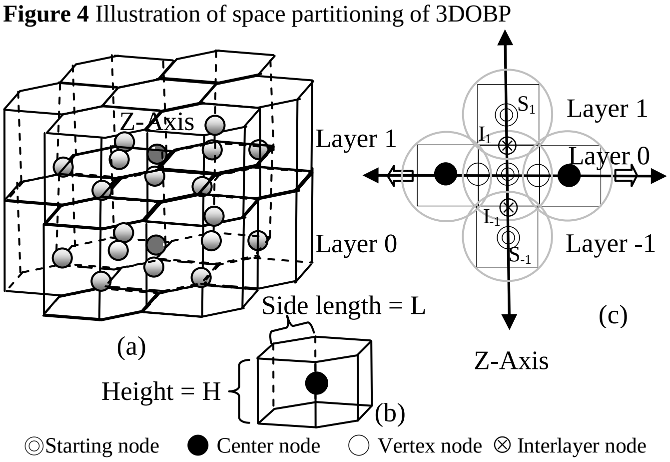
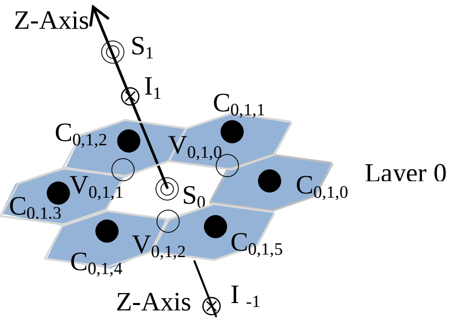


Figure 5 Illustration of activation of starting nodes along the Z-axis and activation of center nodes in the layer 0



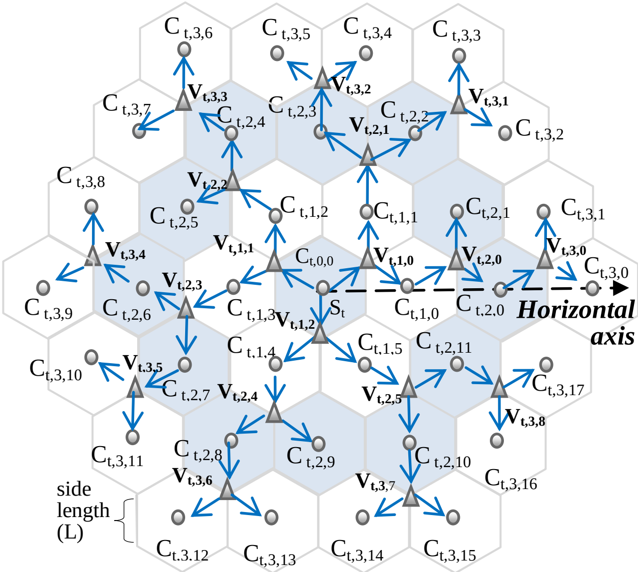
How to activate center nodes in a layer

As shown in Fig. 5, each layer forms hexagonal rings that can fully cover a layer when the rings expand. Hence, if all center nodes are activated to forward the broadcast packet, then all the nodes in the layer can receive the broadcast packet. Note that not all vertex nodes need to be activated. They are activated not for the purpose of covering the layer but for the purpose of forwarding the broadcast packet to center nodes. As we will show, a center node can rely on a vertex node to deliver packets to two center nodes. Thus, the number of activated vertex nodes is just half the number of activated center nodes.

As shown in Fig. 5, the source node S_0 in layer 0 first activates three vertex nodes $V_{0,1,0}$, $V_{0,1,1}$, $V_{0,1,2}$ and two interlayer nodes I_1, I_{-1}. One vertex node then activates two center nodes. For example, the vertex node $V_{0,1,0}$ activates two center nodes $C_{0,1,0}$ and $C_{0,1,1}$. Similarly, the vertex nodes $V_{0,1,1}$ and $V_{0,1,2}$ activate center nodes ($C_{0,1,2}$, $C_{0,1,3}$) and ($C_{0,1,4}$, $C_{0,1,5}$), respectively. We have mentioned how S_0 activates other starting nodes in different layers with the help of interlayer nodes. Below, we describe how S_0 activates all center nodes in a layer with the help of hexagon rings.

The hexagon rings have only one hexagon in the central (level-0) ring, and have six hexagons in the level-1 ring, and so on. In general, there are $6k$ hexagons in the level-k ring. A hexagon center in layer t and in the level-k ring is denoted as $C_{t,k,i}$, where i is an index ranging from 0 to $6k$-1. Centers indexed by 0 lie on the horizontal axis staring from S_t towards right, while other centers are indexed counterclockwise (see Fig. 6).

Figure 6 Illustration of hexagon rings (in layer t) of 3DOBP



The relative location $LC_{t,k,i}$ of $C_{t,k,i}$ relative to S_t can be derived handily by a *geometric mapping* $M(C_{t,k,i}) \rightarrow LC_{t,k,i}$. The geometric mapping will be well defined in subsection 3.2. Additionally, the LS_t is $LC_{t,0,0}$ and the LI_t is $(LC_{t-1,0,0}+LC_{t,0,0})/2$ for $t>0$ (on the other hand, LI_t is $(LC_{t+1,0,0}+LC_{t,0,0})/2$ for $t<0$), where LS_t and LI_t respectively denote the relative locations of starting node S_t and interlayer node I_t, which are relative to the absolute location of the source node S.

In 3DOBP, the source node S_0 (associated with $C_{t,0,0}$) should send the broadcast packet and activate six center nodes (associated with $C_{t,1,0},\ldots,C_{t,1,5}$) in the level-1 ring to forward the packet. And each center node associated with $C_{t,k,i}$ in the level-k ring, $k\geq1$, should either activate no node or activate two neighboring center nodes in the next level. Actually, for $k\geq1$, $3(k+1)$ center nodes in the level-k ring need to activate 2 neighboring center nodes in the level-$(k+1)$ ring, while $3(k-1)$ nodes need not to activate any node. For example, all 6 level-1 center nodes need to activate 2 level-2 center nodes, and thus all 12 level-2 center nodes can be activated properly. For another example, 9 (resp., 3) out of 12 level-2 center nodes need to (resp., need not to) activate 2 level-3 center nodes, and thus all 18 level-3 center nodes can be activated properly. We devise a mapping called the *activation target mapping* $T(C_{t,k,i})$ that outputs an empty set or a set $\{C_{t,k+1,w}, C_{t,k+1,w+1}\}$ of two next-level neighboring center nodes for $C_{t,k,i}$, $k \geq 1$, to activate. Note that $C_{t,k+1,w}$ (resp., $C_{t,k+1,w+1}$) must be a neighboring center node of $C_{t,k,i}$; i.e., the associated hexagons of $C_{t,k,i}$ and $C_{t,k+1,w}$ (resp., $C_{t,k+1,w+1}$) must share an edge. The activation target mapping will be well defined in subsection 3.3.

By the node activation process just mentioned, all center nodes can be activated to transmit the packet to cover the entire layer. However, since two center nodes cannot communicate with each other directly, we need intermediate nodes between them for relaying the packet. 3DOBP chooses vertex nodes (i.e., the node nearest to a hexagon vertex) as the intermediate nodes to take the advantage that a vertex node can reach two center nodes (e.g., $V_{t,1,0}$ can reach $C_{t,1,0}$ and $C_{t,1,1}$). In 3DOBP, S takes 3 vertex nodes associated with $V_{t,1,0}$, $V_{t,1,1}$, and $V_{t,1,2}$ as intermediate nodes, while the other center node associated with $C_{t,k,i}$ takes only 1 (or 0) vertex node associated with $V_{t,k+1,i}$. The relative location $LV_{t,k,i}$ of $V_{t,k,i}$, $k>1$, which is relative to S, can be derived by computing the location of the center of $C_{t,k-1,i}$, $C_{t,k,w}$, and $C_{t,k,w+1}$ if $C_{t,k-1,i}$ should activate $C_{t,k,w}$ and $C_{t,k,w+1}$ (i.e., $T(C_{t,k-1,i})=\{C_{t,k,w}, C_{t,k,w+1}\}$). Note that for $k=1$, $LV_{t,1,i}$ is the location of the center of S, $C_{t,1,2i}$, and $C_{t,1,2i+1}$ for $i=0,1,2$.

The broadcast packet of 3DOBP is of the format P(LS, F, FA), where LS is the absolute location of the source node, and F is the set of relative locations of intended forwarding nodes in the next-level ring. And, FA is the set of relative locations of intended interlayer (or starting) nodes in the neighboring layers. Note that each packet is sent along with a unique packet ID so that a node can decide if the packet has ever been received. Also note that the relative locations are delivered along with the indexes of center nodes or vertex nodes. That is, when a location $LC_{t,k,i}$ or $LV_{t,k,i}$ is delivered, the indexes t, k and i are also delivered in the packet. Those indexes are very important for a node to calculate the relative locations of intended forwarding nodes by the activation target mapping and the geometric mapping.

3D Optimized Broadcast Protocol (3DOBP)

The step for the source node S to broadcast a packet P

1. S sends the packet P(LS, F, FA) with $F=\{LV_{0,1,0}, LV_{0,1,1}, LV_{0,1,2}\}$, $FA=\{LI_1, LI_{-1}\}$.

Steps for other node X receiving P(LS, F, FA):

1. If X receives P for the first time, it registers P. Otherwise, it drops P and stops.
2. If X is not a node nearest to a location in F or FA, it stops.
3. If X is nearest to a center node associated with $C_{t,k,i}$ of a location in F and $T(C_{t,k,i}) \neq \emptyset$, then X sends P(LS, F′) and stops, where $F'=\{LV_{t,k,i}\}$.
4. If X is nearest to a vertex node associated with $V_{t,k,i}$ of a location in F, X sends P(LS, F′, ∅) and stops. Indeed, X can calculate $T(C_{t,k-1,i})=\{C_{t,k,w}, C_{t,k,w+1}\}$ based on indexes k, i and then set $F'=\{LC_{t,k,w}, LC_{t,k,w+1}\}$.
5. If X is nearest to an interlayer node associated with I_t of a location in FA, X sends P(LS, ∅, FA′) and stops, where :

$$FA' = \begin{cases} \{LS_{t+1}\}, \text{if } t > 0 \ \ (i.e., updirection\ activation) \\ \{LS_{t-1}\}, \text{if } t < 0 \ \ (i.e., downdirection\ activation) \end{cases}$$

6. If X is nearest to a starting node associated with S_t of a location in FA, X sends P(LS, F′, FA′) and stops, where : $F'=\{LV_{t,1,0}, LV_{t,1,1}, LV_{t,1,2}\}$

$$FA' = \begin{cases} \{LI_{t+1}\}, \text{if } t > 0 \ (i.e., updirection\ activation) \\ \{LI_{t-1}\}, \text{if } t < 0 \ (i.e., downdirection\ activation) \end{cases}$$

The article (Paruchuri, 2006) proposes two mechanisms to determine whether a node X is the one nearest to a given location. The first mechanism is to make nodes periodically exchange location information with neighboring nodes so that each node can properly elect the node nearest to the given location. The second mechanism is to enforce a backoff timer which is inversely proportional to the distance between a node's location and the given location. The node nearest to the given location thus has the shortest backoff timer and will earliest issue a response, which in turn prohibits other nodes from responding. When node density is sufficiently high, it is usual that only the node closest to the selected location would retransmit the packet. 3DOBP adopts the second mechanism to determine whether a node is the one nearest to a given location. In this way, a node does not need to exchange location information periodically; a node needs only its own location information for 3DOBP to run properly.

3.2 Geometric Mapping

The geometric mapping $M(C_{t,k,i})$ maps a hexagon center node $C_{t,k,i}$ to a location $L_{t,k,i}$ relative to the source node S. Let $Z_{t,0,0}$ denote the location relative to S in the t-layer. As shown in Fig. 7, each hexagon ring can be partitioned into six sectors, indexed by 0,..,5, with each sector having k hexagon centers in the level-k ring. Let $Z_{t,k,q}$ denote the location relative to S of the first hexagon center in the sector q of the level-k hexagon ring in the layer t (e.g., $Z_{1,2,0}$ is the location of the hexagon center on the horizontal line from S towards right in the layer 1). The above relationship is defined as $Z_{t,k,q} = \left(\mathrm{H} \cdot t, k\sqrt{3}\mathrm{L} \cdot \cos(q \cdot 60°), k\sqrt{3}\mathrm{L} \cdot \sin q{\cdot}60°\right.$ for $q=0,..,5$, where L is the hexagon side length determined by Eq. (4), and H is the height of hexagon prism determined by Eq. (5).

Since each sector has k hexagon centers, we can figure out that hexagon center $C_{t,k,i}$ is within sector q, where $q = \lfloor i/k \rfloor$. Now we can define the geometric mapping $M(C_{t,k,i})$ as follows. Note that '+' represents the vector addition operator in the mapping and the following location calculations.

Let $q=\lfloor i/k \rfloor$.

If i is a multiple of k, $M(C_{t,k,i})=Z_{t,k,q}$.
Otherwise, $M(C_{t,k,i})$

$$= \begin{cases} Z_{t,k,0} + (0, -i\dfrac{\sqrt{3}\mathrm{L}}{2}, i\dfrac{3\mathrm{L}}{2}), \text{if } q = 0 \\ Z_{t,k,1} + (0, (k-i)\sqrt{3}\mathrm{L}, 0), \text{if } q = 1 \\ Z_{t,k,2} + (0, (q \cdot k - i)\dfrac{\sqrt{3}\mathrm{L}}{2}, (q \cdot k - i)\dfrac{3\mathrm{L}}{2}), \text{if } q = 2 \\ Z_{t,k,3} + (0, (i - q \cdot k)\dfrac{\sqrt{3}\mathrm{L}}{2}, (q \cdot k - i)\dfrac{3\mathrm{L}}{2}), \text{if } q = 3 \\ Z_{t,k,4} + (0, (i - q \cdot k)\sqrt{3}\mathrm{L}, 0), \text{if } q = 4 \\ Z_{t,k,5} + \left(0, (i - q \cdot k)\dfrac{\sqrt{3}\mathrm{L}}{2}, (i - q \cdot k)\dfrac{3\mathrm{L}}{2}\right), \text{if } q = 5 \end{cases}$$

In Fig. 7, we illustrate the above mapping by two examples. The first example is about $M(C_{0,2,1})$. Since $q=\lfloor 1/2 \rfloor=0$, we calculate $Z_{0,2,0}=(0, 2\sqrt{3}\,\mathrm{L}\cdot\cos(0), 2\sqrt{3}\,\mathrm{L}\cdot\sin(0))=(0,2\sqrt{3}\,\mathrm{L},0)$. We then have

$M(C_{0,2,1})=Z_{0,2,0}+(0,\ -\frac{\sqrt{3}L}{2}, \frac{3L}{2})$. The second example is about $M(C_{0,2,7})$. Since $q=\lfloor 7/2 \rfloor=3$, we calculate $Z_{0,2,3}=(0, 2\sqrt{3}\ L\cdot\cos(180°), 2\sqrt{3}\ L\cdot\sin(180°))=(0, -2\sqrt{3}L, 0)$. We then have $M(C_{0,2,7})=Z_{0,2,3}+(0, \frac{\sqrt{3}L}{2}, -\frac{3L}{2})$.

3.3. Activation Target Mapping

In this subsection we present the activation target mapping $T(C_{t,k,i})$. The input of $T(C_{t,k,i})$ is a center node $C_{t,k,i}$ for $k\geq 1$. $T(C_{t,k,i})$ is to find two next-level neighboring center nodes $C_{t,k+1,w}$ and $C_{t,k+1,w+1}$ of $C_{t,k,i}$, where w is even. If such neighboring nodes exit, the output of $T(C_{t,k,i})$ is $\{C_{t,k+1,w}, C_{t,k+1,w+1}\}$; otherwise, the output is an empty set. For example, $T(C_{0,1,0})=\{C_{0,2,0}, C_{0,2,1}\}$ since $C_{t,k+1,w}$ is $C_{0,2,0}$ and $C_{t,k+1,w+1}$ is $C_{0,2,1}$ with w=0. For another example, $T(C_{0,2,1})=\emptyset$, since the index w of the neighboring center nodes ($C_{0,3,1}$ and $C_{0,3,2}$) of $C_{0,2,1}$ is odd.

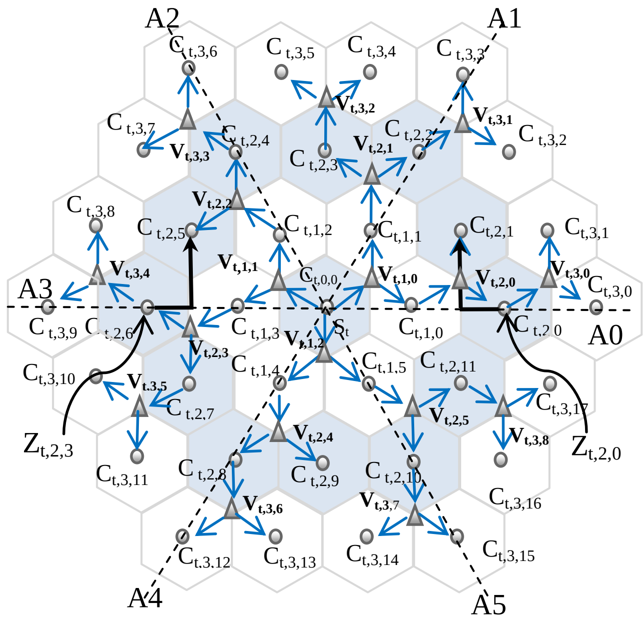
As shown in Fig. 7, each hexagon ring can be partitioned into six sectors, indexed by 0,..,5, each having a starting axis (i.e., A0, ,…, A5). Let $q=\lfloor i/k \rfloor$ denote the index of the sector in which $C_{t,k,i}$ resides. $T(C_{t,k,i})$ is defined as follows.

$$T(C_{t,k,i})=\begin{cases} \{C_{t,k+1,i}, C_{t,k+1,i+1}\}, & \text{if } q=0 \text{ and } i \text{ is even} \\ \{C_{t,k+1,i+1}, C_{t,k+1,i+2}\}, & \text{if } q=1 \text{ and } i \text{ is odd} \\ \{C_{t,k+1,i}, C_{t,k+1,i+1}\}, & \text{if } q=1, (i \bmod k)=0 \text{ and } i \text{ is even} \\ \{C_{t,k+1,i+2}, C_{t,k+1,i+3}\}, & \text{if } q=2 \text{ and } i \text{ is even} \\ \{C_{t,k+1,i+3}, C_{t,k+1,i+4}\}, & \text{if } q=3 \text{ and } i \text{ is odd} \\ \{C_{t,k+1,i+2}, C_{t,k+1,i+3}\}, & \text{if } q=3, (i \bmod k)=0 \text{ and } i \text{ is even} \\ \{C_{t,k+1,i+4}, C_{t,k+1,i+5}\}, & \text{if } q=4 \text{ and } i \text{ is even} \\ \{C_{t,k+1,i+5}, C_{t,k+1,i+6}\}, & \text{if } q=5 \text{ and } i \text{ is odd} \\ \{C_{t,k+1,i+4}, C_{t,k+1,i+5}\}, & \text{if } q=5, (i \bmod k)=0 \text{ and } i \text{ is even} \\ \emptyset, & \text{otherwise} \end{cases}$$

The definition of $T(C_{t,k,i})$ contains 10 cases of different conditions. The first case (q=0 and i is even) drives the center node $C_{t,k,i}$ to activate two neighboring center nodes $C_{t,k+1,i}$ and $C_{t,k+1,i+1}$ in the next level, only if $C_{t,k,i}$ is in sector 0 and i is even. The second case (q=1 and i is odd) drives $C_{t,k,i}$ to activate $C_{t,k+1,i+1}$ and $C_{t,k+1,i+2}$, only if $C_{t,k,i}$ is in sector 1 and i is odd. The third case (q=1, (i mod k)=0 and i is even) drives $C_{t,k,i}$ to activate $C_{t,k+1,i}$ and $C_{t,k+1,i+1}$, only if $C_{t,k,i}$ is in sector 1, $C_{t,k,i}$ is on the starting axis of sector 1 (i.e. A1), and i is even. Similarly, the fourth, ..., and the ninth cases drive $C_{t,k,i}$ in the sectors 2, …, and 5 to activate two center nodes for specific conditions. The last case drives $C_{t,k,i}$ not to active any node, only if none of the first nine conditions is satisfied.

In Fig. 7, we illustrate $T(C_{t,k,i})$ by three examples. The first example is about $T(C_{0,2,0})$. Let $q=\lfloor 0/2 \rfloor=0$. Since q is 0 and i (=0) is even, we have $T(C_{0,2,0})=\{C_{0,3,0}, C_{0,3,1}\}$. The second example is about $T(C_{0,2,5})$. Let $q=\lfloor 5/2 \rfloor=2$. Since q is 2 and i (=5) is not even, we have $T(C_{0,2,5})=\emptyset$, which means the node $C_{0,2,5}$ needs not to activate any node. The third example is about $T(C_{0,2,2})$. Let $q=\lfloor 2/2 \rfloor=1$. Since q is 1, i (=2) is even, and i is a multiple of k (=2), we have $T(C_{0,2,2})=\{C_{0,3,2}, C_{0,3,3}\}$.

Figure 7 The illustration of the Geometric Mapping and Activation Target Mapping



4 Performance Analysis

In this section, we first derive the theoretical upper bound of transmission efficiency for 3D wireless networks. We then analyze and compare the transmission efficiency of the proposed 3DOBP and the 3D broadcast protocols based on filling space by polyhedrons: cubes, hexagonal prisms, rhombic dodecahedrons, and truncated octahedrons.

4.1 Transmission Efficiency Upper Bound

The transmission efficiency is defined as the effective transmission region over the sum of the transmission regions. We use two spheres A and B of radius R in Fig. 8 to derive the transmission efficiency upper bound.

According to (Weisstein, 2011), the intersected volume of two spheres is as follows:

$$V_{inter}(R,d)=\pi(4R+d)\,(2R-d)^2/12, \text{ for } d \leq 2R \qquad (6)$$

In Eq. (6), R is the radius of sphere and d is the distance of two spheres.

To minimize the intersected volume, while keeping the two sphere center within distance R is d=R. Hence, the upper bound of transmission efficiency (TE) in the 3D space is as follows:

$$TE(R)=\frac{2V_{sphere}(R)-V_{inter}(R,R)}{2V_{sphere}(R)} \qquad (7)$$

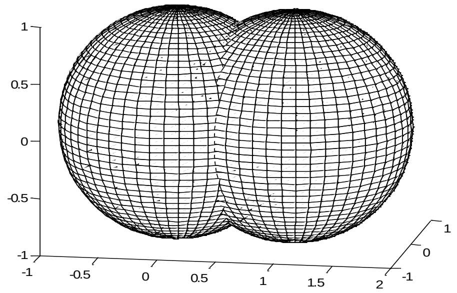
In Eq. (7), V_{sphere} is the volume of sphere with radius R, which is $(4/3)\pi R^3$.

Letting R=1, we have the upper bound of transmission efficiency in 3D as follows:

$$TE = 1-\frac{(5/12)\pi}{2(4/3)\pi}=\frac{27}{32}=0.84375 \qquad (8)$$

Thus, we have that the transmission efficiency upper bound is 0.84375.

Figure 8 Intersected spheres, where center of sphere A on the left is at (0,0,0) and sphere B on the right is at (1,0,0), and the distance between two centers is one unit distance.



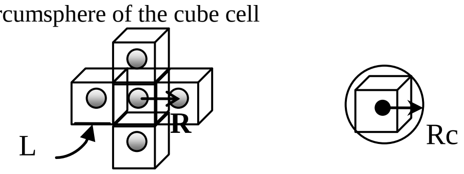
4.2 Transmission Efficiency of Cubes

We assume a cube is with the side length L. The center of each cube is located by a node of transmission radius R. We can form a circumsphere for a cube, where the radius of circumsphere is Rc. The relationship between the side length and the radius of circumsphere is: L= $2\mathrm{Rc}/\sqrt{3}$. The distance between two neighboring cube centers should be within R. Thus, R=L=2Rc/$\sqrt{3}$.

For a set of N nodes with the cube arrangement, the transmission efficiency is:

$$\frac{N\cdot \mathrm{L}^3}{N\cdot\pi(2\mathrm{Rc}/\sqrt{3})^3\cdot 4/3} = \frac{8\mathrm{Rc}^3/3\sqrt{3}}{\pi 8\mathrm{Rc}^3/3\sqrt{3}\cdot 4/3} = \frac{3}{4\pi} = 0.238732 \qquad (9)$$

Figure 9 Illustration of the cube arrangement, where L is the side length, R is the transmission radius, and Rc is the radius of the circumsphere of the cube cell



4.3 Transmission Efficiency of Hexagonal Prisms

We assume a hexagonal prism is with side length L and height H, and that the center of each hexagonal prism is located by a node with transmission radius R. By Eq. (2), the hexagon prism has the maximal volume when H=$\sqrt{2}$L. We have the volume of hexagonal prism (HPV) is $\frac{3\sqrt{3}}{2}\mathrm{L}^2\cdot \mathrm{L}\sqrt{2} = \frac{3\sqrt{3}}{\sqrt{2}}\mathrm{L}^3$. Moreover, we set the radius Rc of the circumsphere of the hexagonal prism as follows:

$$\mathrm{Rc} = \sqrt{\mathrm{L}^2 + \frac{\mathrm{H}^2}{4}} = \sqrt{\mathrm{L}^2 + \frac{\mathrm{L}^2}{2}} = \sqrt{\frac{3\mathrm{L}^2}{2}} = \mathrm{L}\sqrt{\frac{3}{2}} \qquad (10)$$

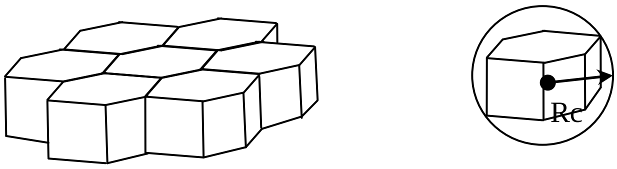
And, the relationship between the transmission radius and the circumsphere radius is R=$\sqrt{2}$ Rc. We have the following equation:

$$\mathrm{R}=\sqrt{2}\,\mathrm{Rc}=\sqrt{2}\,\mathrm{L}\sqrt{\frac{3}{2}} = \sqrt{3}\,\mathrm{L} \qquad (11)$$

For a set of N nodes with hexagonal prism arrangement, the transmission efficiency is as follows:

$$\frac{N\cdot \mathrm{HPV}}{N\cdot\pi \mathrm{R}^3\cdot\frac{4}{3}} = \frac{\frac{3\sqrt{3}}{\sqrt{2}}\mathrm{L}^3}{\pi\cdot(\sqrt{3}\mathrm{L})^3\cdot\frac{4}{3}} = \frac{\frac{3\sqrt{3}}{\sqrt{2}}}{\pi\cdot(\sqrt{3})^3\cdot\frac{4}{3}} = \frac{3}{4\sqrt{2}\pi} = 0.168809 \qquad (12)$$

Figure 10 Illustration of the hexagonal prism arrangement



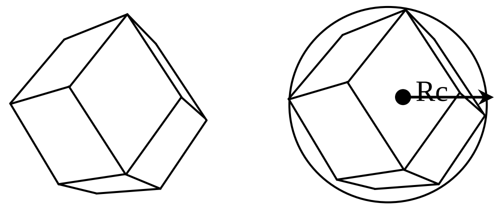
4. 4 Transmission Efficiency of Rhombic Dodecahedrons

A rhombic dodecahedron can be constructed by two cubes of the length L. Based on the results of (Alam and Hass, 2006), we have the following settings. We can construct a circumsphere for a rhombic dodecahedron. The radius Rc of the circumsphere is L. The volume RDV of a rhombic dodecahedron is $2\mathrm{L}^3$. The transmission radius R of a rhombic dodecahedron center node is $\sqrt{2}$ Rc. Since Rc=L, we have R=$\sqrt{2}$ L.

For a set of N nodes with rhombic dodecahedron arrangement, the transmission efficiency is:

$$\frac{\mathrm{N}\cdot\mathrm{RDV}}{\mathrm{N}\cdot\pi\mathrm{R}^3\cdot 4/3} = \frac{2\mathrm{L}^3}{\pi\cdot(\sqrt{2}\mathrm{L})^3\cdot 4/3} = \frac{1}{\pi\cdot\sqrt{2}\cdot 4/3} = \frac{3}{4\sqrt{2}\pi} = 0.168809 \qquad (13)$$

Figure 11 Illustration of the rhombic dodecahedron cell



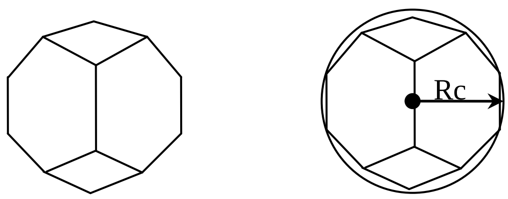
4. 5 Transmission Efficiency of Truncated Octahedrons

We assume that a truncated octahedron is of side length L and the center of each truncated octahedron is located by a node whose transmission radius is R. Let Rc be the radius of the circumsphere of a truncated octahedron. Based on the results of (Alam and Hass, 2006), we set Rc=L$\sqrt{10}$/2 and R=4Rc/$\sqrt{5}$. Then we have R=$\frac{4}{\sqrt{5}}\frac{\mathrm{L}\sqrt{10}}{2}$=2L$\sqrt{2}$.

For a set of N nodes with truncated octahedron arrangement, the transmission efficiency is:

$$\frac{\mathrm{N}\cdot 8\sqrt{2}\mathrm{L}^3}{\mathrm{N}\cdot\pi(2\mathrm{L}\sqrt{2})^3\cdot 4/3} = \frac{\mathrm{N}\cdot 8\sqrt{2}}{\mathrm{N}\cdot\pi(2\sqrt{2})^3\cdot 4/3} = \frac{3}{8\pi} = 0.119366 \qquad (14)$$

Figure 12 Illustration of the truncated octahedron cell



4.6 Transmission Efficiency of 3DOBP

We assume a hexagonal prism is with side length L and height H, and that the center of each hexagonal prism is located by a node with transmission radius R. By Eqs. (4) and (5), we have $L=R/\sqrt{3/2}$ and $H=2R/\sqrt{3}$. The volume HPV of a hexagonal prism is:

$$\mathrm{HPV} = \frac{3\sqrt{3}}{2}\mathrm{L}^2 \cdot \mathrm{H} = \frac{3\sqrt{3}}{2}\mathrm{L}^2 \cdot \mathrm{L}\sqrt{2} = \frac{3\sqrt{3}}{\sqrt{2}}\mathrm{L}^3 \qquad (15)$$

With the help of hexagon rings, in 3DOBP, all center nodes suffice to cover a layer of the space. We assume the number of center nodes is N_c and the number of N_v of vertex nodes to be activated to forward packets to the N_c center nodes. As shown earlier, there are 6*k* hexagons in the level-*k* hexagon ring. Assuming a layer is covered by the level-J hexagon ring with all inner hexagon rings (rings of level 0, level 1,…, to level (J-1)) included, we have

$$N_c = 1 + \sum_{k=1}^{J} 6k = 1 + 3J(J+1) \qquad (16)$$

$$N_v = \frac{1}{2}\sum_{k=1}^{J} 6k = \frac{3}{2}J(J+1) \qquad (17)$$

To simplify the calculation, we assume the number of interlayer nodes is ignorable. The transmission efficiency (TE) of the 3DOBP is

$$\begin{aligned}\mathrm{TE} &= \lim_{J\to\infty} \frac{\mathrm{Nc}\cdot\mathrm{HPV}}{(\mathrm{Nc}+\mathrm{Nv})\cdot\pi\mathrm{R}^3\cdot\frac{4}{3}} = \lim_{J\to\infty} \frac{(1+3J(J+1))\cdot\mathrm{HPV}}{\left(1+\frac{3}{2}(3J(J+1))\right)\cdot\pi\mathrm{R}^3\cdot\frac{4}{3}} \\ &= \frac{3}{4}\lim_{J\to\infty} \frac{(1+3J(J+1))\cdot\mathrm{HPV}}{\left(1+\frac{3}{2}(3J(J+1))\right)\cdot\pi\mathrm{R}^3} \\ &\xrightarrow{L'Hospital\ Rule} \frac{3}{4}\frac{(6)\cdot\mathrm{HPV}}{(9)\cdot\pi\mathrm{R}^3} = \frac{1}{\pi}\end{aligned} \qquad (18)$$

In Eq. (18), the famous L'Hospital Rule (Zill et al., 2009) is used to derive the limit of TE of 3DOBP. Since $J\to\infty$ and the numerator and denominator are both differentiable, the limit value is $1/\pi$.

4.7 Transmission Efficiency Comparison

In this subsection, we compare 3DOBP with other 3D broadcast protocols based on filling space by polyhedrons like cubes, hexagonal prisms (Alam and Hass, 2006; Decayeux and Seme, 2004), rhombic dodecahedrons (Alam and Hass, 2006), and truncated octahedrons (Alam and Hass, 2006; Carle et al., 2001) in terms of transmission efficiency. Note that the comparisons do not include the 3D broadcast protocol AACP proposed in (Durresi et., 2006), since the network space is not fully covered even when node density is sufficiently high. The comparison results are summarized in the Table 1. We can observe that 3DOBP is with the highest transmission efficiency. We also compare the number of required nodes in different approaches. The comparison results are summarized in the Table 2.

Table 1 Comparison of Transmission Efficiency

Approach	*Transmission efficiency*
Cube	$3/4\pi \approx 0.238732$
Hexagonal Prism	$3/(4\sqrt{2}\,\pi) \approx 0.168809$
Rhombic Dodecahedron	$3/(4\sqrt{2}\,\pi) \approx 0.168809$
Truncated Octahedron	$3/8\pi \approx 0.119366$
3DOBP	$1/\pi \approx 0.31831$

Table 2 Comparison of Required Nodes

Approach	*Number of nodes Compared to 3DOBP*
Cube	4/3
Hexagonal Prism	$4\sqrt{2}/3$
Rhombic Dodecahedron	$4\sqrt{2}/3$
Truncated Octahedron	8/3
3DOBP	1

5 Simulation

In this section, we evaluate by simulation the performance of 3DOBP and the 3D broadcast protocols based on filling space by polyhedrons, such as cubes, hexagonal prisms, rhombic dodecahedrons, and truncated octahedrons. We develop a simulator using Matlab (Matlab 2011). We first establish a set of *backbone nodes* to form a "perfect" broadcast backbone for broadcasting in 3D space for each 3D broadcast protocol. For example, for the 3DOBP protocol, the set of backbone nodes is controlled by (*t*, *k*, *i*) indexes, where index *t* denotes the *t*-th layer along the *z*-axis, index *k* denotes the *k*-th hexagonal ring, and index *i* denotes the *i*-th node in the hexagonal ring. The pseudo code to establish the backbone nodes is as follows. In the pseudo code, parameter B, called *network size* later, is used to control the size of the broadcast backbone. Moreover, $M(C_{t,k,i})$ represents the location (relative to the origin) of a center backbone node and $VM(C_{t,k,i})$ represents the location (relative to the origin) of a vertex backbone node. DeployNode($M(C_{t,k,i})$) is a function to put a backbone node in the exact position indicated by $M(C_{t,k,i})$. And DeployInterlayNode(*t*) is to put an interlayer backbone node between layers *t* and *t*+1 right above or below the origin. Note that any two adjacent layers are of the distance $H=2\frac{R}{\sqrt{3}}$.

```
FOR t= -B, -B+1, …, B  {
   FOR k= 0, 1, …, B
        FOR i= 0, 1, …, k*6-1 {
             DeployNode( M(C_t,k,i) )
             IF T(C_t,k,i) ≠ ∅ THEN
                DeployNode( VM(C_t,k,i) )
        }
   DeployInterlayerNode(t)
}
```

For other broadcast protocols, the value of each of *u*, *v*, and *w* in Table 3, which is based on (Alam & Hass, 2006), is assigned as –B, –B+1, …, 0, 1, …, and B to establish the backbone nodes. As shown later, the value of B may be 2, 3, …, or 8 for our simulation cases. After establishing the broadcast backbone, we establish a set of *test nodes* within a cube region much larger than the space of the union of all backbone nodes' broadcast space for measuring the protocol performance. The test node is established every R/5 distance along the X, Y and Z axes. The source node is assumed to be located at the origin to issue the broadcast packet, and all backbone nodes are assumed to forward the broadcast packet on receiving it for the first time. A *covered test node* is a test node that can receive the packet sent from the source node or the backbone nodes properly.

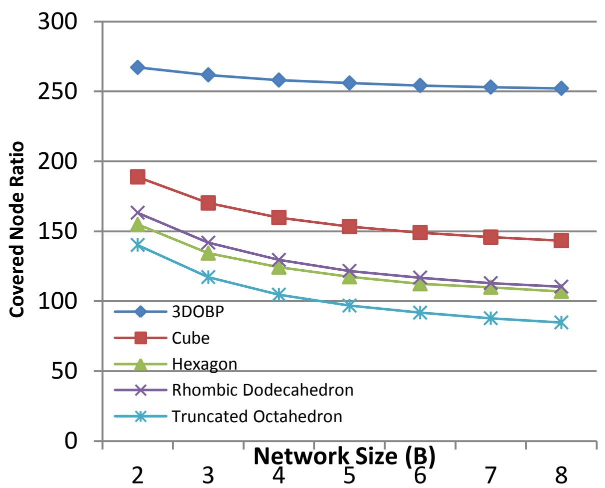
Table 3 Deployment Structures

Approach	*Location*
Cube	$(u{\cdot}R, v{\cdot}R, w{\cdot}R)$
Hexagonal Prism	$(\frac{\sqrt{3}\,u}{2}R, \frac{u+2v}{2}R, \frac{2w}{\sqrt{6}}R)$
Rhombic Dodecahedron	$(\frac{2u+w}{2}R, \frac{2v+w}{2}R, \frac{w}{\sqrt{2}}R)$
Truncated Octahedron	$(\frac{2u+w}{2}R, \frac{2v+w}{2}R, \frac{\sqrt{2}\,w}{2}R)$

■ Evaluation of Transmission Efficiency

We can evaluate transmission efficiency by calculating the ratio of the number of covered test nodes to the number of backbone nodes. As shown in Fig. 13, 3DOBP has the highest covered node ratio, complying with the analysis results showing 3DOBP has the highest transmission efficiency.

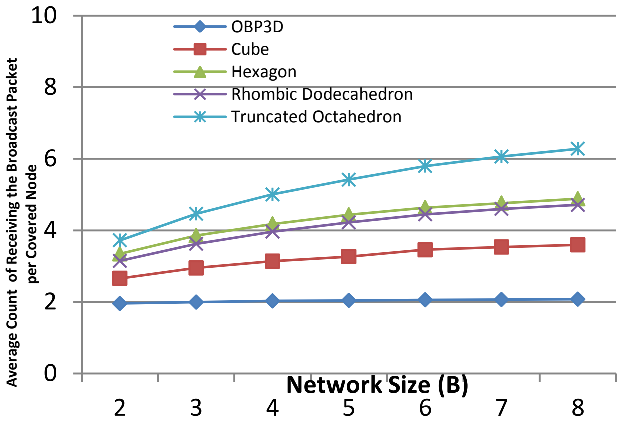
Figure 13 Covered node ratios of different 3D broadcast protocols



■ Evaluation of Redundancy and Collision Degree

The average count of receiving the broadcast packet per covered node can be used to evaluate the redundancy and the collision degree of broadcasting. The larger the average count is, the higher the collision degree and the transmission redundancy are. As shown in Fig. 14, 3DOBP has the smallest average count among all protocols, which implies 3DOBP has the lowest transmission redundancy and collision degree.

Figure 14 Average count of receiving the broadcast packet per covered node for different broadcast protocols



6 Conclusion

Broadcasting is a fundamental operation in the wireless network to disseminate data and/or control signals. In this paper, we have generalized the definition of transmission efficiency for 3D wireless networks and shown that the theoretical upper bound of transmission efficiency of 3D broadcast protocols is 0.84. We have further proposed an optimized broadcast protocol, called 3D optimized broadcast protocol (3DOBP), by partitioning the 3D space into multi-layer hexagonal prisms of a hexagon ring pattern in each layer. As we have shown, the transmission efficiency of the proposed protocol can reach $1/\pi$, which is better than those of other 3D broadcast protocols based on polyhedron-filling schemes using cubes, hexagon prisms, rhombic dodecahedrons, and truncated octahedrons. We conclude that 3DOBP is very efficient for broadcasting in 3D wireless networks.

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