國立中央大學資訊工程學系 105 學年度第一學期博士班資格考試題紙

科目： **演 算 法** (Algorithms)

1. Below is the Recursive Fibonacci Algorithm (RFA). An integer n, $n \geq 1$, is inputted into the algorithm for the algorithm to output the nth item of the Fibonacci series. Please answer the following questions.
   (a) What is the 10th item of the Fibonacci series? (5%)
   (b) Analyze the time complexity of RFA. (15%)

   Algorithm RFA(n)
   Input: integer n, $n \geq 1$
   Output: the nth item of the Fibonacci series

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if n=1 or n=2 then
    return 1
else
    a ← RFA(n-2)
    b ← RFA(n-1)
    return a+b
```

2. The 0/1 knapsack problem is described as follows. Given the capacity m of a knapsack and n objects whose weights are $w_1,\ldots,w_n$ and whose profits are $p_1,\ldots,p_n$, find the largest value of $\sum_{1 \leq i \leq n} p_i x_i$ by assigning either 0 or 1 to $x_1,\ldots,x_n$ under the constraint $\sum_{1 \leq i \leq n} w_i x_i \leq m$, where $w_1,\ldots,w_n$ and $p_1,..,,p_n$ are positive integers. Write a dynamic programming algorithm to solve the 0/1 knapsack problem with the time complexity O($n \times m$). You should show that the time complexity of your algorithm is indeed O($n \times m$). (20%)

3. Given a fact that problem X is NP-hard, how can we prove that problem Y is also NP-hard by taking advantage of polynomial-time reduction and the fact? (10%)

4. Given a set of 2n−1, n ≥ 1, positive integers (not necessarily distinct) S, it is known that we can always choose n numbers from S such that the sum of these n numbers is divisible by n, i.e. the sum can be divided by n with no remainder. Design a polynomial time algorithm to find such n numbers from S. (20%)

5. Given a positive weighted di-graph G, and a pair of vertices u and v in G, we want to find a simple path (i.e. all vertices in the path are distinct) from u to v with maximum length. (a) Show that this problem is NP-Hard (15%) (b) However, if G contains no cycle, then show this problem can be solved efficiently. (15%)