1. 30% Consider the linear system

\[
\begin{array}{ccc}
  x + y - z &=& -2 \\
 3x - 5y + 13z &=& 18 \\
x - 2y + 5z &=& k
\end{array}
\]

where \( k \) is an arbitrary number.

a. For which values(s) of \( k \) does this system have one or infinitely many solutions?

b. For each value of \( k \) you found in part a, how many solutions does the system have?

c. Find all solutions for each value of \( k \).

2. 20% Find all solutions \( x_1, x_2, x_3 \) of the equation

\[ \vec{b} = x_1 \vec{v}_1 + x_2 \vec{v}_2 + x_3 \vec{v}_3 \]

where

\[ \vec{b} = \begin{bmatrix} -8 \\ -1 \\ 2 \\ 15 \end{bmatrix}, \vec{v}_1 = \begin{bmatrix} 1 \\ 4 \\ 7 \\ 5 \end{bmatrix}, \vec{v}_2 = \begin{bmatrix} 2 \\ 5 \\ 8 \\ 3 \end{bmatrix}, \vec{v}_3 = \begin{bmatrix} 4 \\ 6 \\ 9 \\ 1 \end{bmatrix}. \]

3. 20% Compute the products \( A \vec{x} \) for

\[
\begin{bmatrix}
1 & 2 & 3 \\
2 & 3 & 4
\end{bmatrix}
\begin{bmatrix}
-1 \\
2 \\
1
\end{bmatrix}
\]

4. 20% True or False?

a. Consider the system \( A \vec{x} = \vec{b} \), where \( A \) is an \( n \times n \) matrix. This system has a unique solution if and only if \( \text{rank}(A) = n \).

b. Matrix \( \begin{bmatrix}
1 & 2 & 0 \\
0 & 0 & 1 \\
0 & 0 & 0
\end{bmatrix} \) is in rref (reduced row-echelon form).

5. 10% Let \( A \) be the \( n \times n \) matrix with all 1’s on the diagonal and all 0’s outside the diagonal. What is \( A \vec{x} \), where \( \vec{x} \) is a vector in \( R^n \)?