1. (10%) By definition, show that $7n^4 + 2n^2 + 6n - 5$ is of $\theta(n^4)$.

2. (20%) Write a prune-and-search algorithm of $O(n)$ time complexity to solve the selection problem: given a set $S$ of $n$ elements, find the $k$-th smallest element of $S$. You should show that your algorithm is of $O(n)$ time complexity.

3. (20%) Write a divide-and-conquer algorithm of $O(n \log n)$ time complexity to solve the closest pair problem: given $n$ points on a plane, find a pair of points with the smallest distance between them. You should show your algorithm is of $O(n \log n)$ time complexity.

4. (30%) Given a weighted undirected graph $G(V, E, w)$, such that each edge $e$ in $E$ is assigned a non-negative weight $w(e)$, the problem Longest-Cycle is to find a longest cycle in $G$ and the problem Shortest-Cycle is to find a shortest cycle in $G$. Decide which problem is polynomial-time solvable and which one is NP-hard. For the problem that is polynomial-time solvable, you should give an algorithm to solve it, and for the problem that is NP-hard, you should describe a known NP-hard problem and then show that the latter is polynomial time reducible to the former.

5. (20%) Show how to count the number of distinct substrings of a string $T$ in time $O(n)$, where the length of $T$ is $n$. Also show how to enumerate one copy of each distinct substring in time proportional to the length of all those strings.