國立中央大學資訊工程學系 109 學年度第一學期博士班資格考試題紙

科目： **演 算 法** (Algorithms) 第一頁 共一頁(page 1 of 1)

1. A string is a sequence of symbols; for example, $X = \langle x_1, x_2, ..., x_m \rangle$ is a string of m symbols x_1, x_2,..., x_m. When we delete 0 or more symbols (not necessarily consecutive) from X, we get a subsequence of X. (a) (15%) Write a dynamic programming algorithm $LCSS(X, Y)$ to calculate the length of the longest common subsequence of $X=\langle x_1, x_2, ..., x_m \rangle$ and $Y=\langle y_1, y_2, ..., y_n \rangle$. (b) (8%) Analyze the time complexity of the *LCSS* algorithm.

2. (15%) Let $S=\{s_1, s_2, ..., s_n\}$ be a non-empty set of n elements. Write an algorithm to select the media of S with the linear time complexity in the worst case.

3. (12%) Suppose problem X has been proven to be an NP-hard problem. Show that how to prove a given problem Y to be NP-hard based on the NP-hardness of problem X.

4. (25%) Let $x_1, x_2, \ldots, x_n$ be a sequence of real numbers (not necessarily positive). Design an algorithm to find a subsequence $x_i, x_{i+1}, \ldots, x_j$ (of consecutive elements) such that the ***product*** of the numbers in it is maximum over all subsequences of consecutive elements. The product of the empty subsequence is defined as 1.

5. (25%) Suppose you have one machine and a set of n jobs to process on that machine. Each job j has a processing time t_j, a profit p_j, and a deadline d_j and the machine can process only one job at a time. If job j is completed by its deadline d_j, you receive a profit p_j, but if it is completed after its deadline, you receive a profit of 0. Give an algorithm to find the schedule that obtains the maximum amount of profit.