國立中央大學資訊工程學系 108 學年度第二學期博士班資格考試題紙

科目： **演 算 法** (Algorithms)

1. A string is a sequence of symbols; for example, $X = <x_1, x_2,..., x_m>$ is a string of m symbols $x_1, x_2,..., x_m$. When we delete 0 or more symbols (not necessarily consecutive) from X, we get a subsequence of X. (a) (14%) Write a dynamic programming algorithm *LCSS*(X, Y) to calculate the length of the longest common subsequence of $X=<x_1, x_2,..., x_m>$ and $Y=<y_1, y_2,..., y_n>$. (b) (8%) Analyze the time complexity of the *LCSS* algorithm. (c) (8%) Let every symbol in X be distinct. Write an algorithm to derive the longest increasing subsequence of X based on the *LCSS* algorithm.

2. Let $S=\{s_1, s_2,..., s_n\}$ be a non-empty set of n elements. Write an algorithm to select the median of S with the linear time complexity in the worst case. (20%)

3. Given a set S of n real numbers, and another real number M, we want to determine whether or not there exist 3 numbers in S whose sum is exactly M. The algorithm of testing all possible 3 numbers in S will take $O(n^3)$ time and it is unacceptable.

a) (10%) Design a more efficient algorithm to solve this problem. Analyze the time complexity of your algorithm.

b) (5%) Consider the following similar problem: Given a set S of n real numbers, another real number M, and an integer k, we want to determine whether or not there exist k numbers in S whose sum is exactly M. Show that this problem is NP-Complete.

c) (10%) If M is small enough, then it is possible to solve the above problem efficiently in $O(nkM)$ time. Design such an algorithm.

4. Consider the problem of finding minimum spanning tree (MST): Given a weighted undirected graph, find a spanning tree with the best (minimum) cost, where the cost of a spanning tree is the sum of the weights of its edges.

a) (15%) Describe an efficient algorithm for solving this problem. You should also analyze the time complexity of this algorithm and describe the data structure used by the algorithm.

b) (10%) Now assume that we also want to know a spanning tree with the second best cost (if there is any which may be same as the best cost). For example, consider the graph with vertex set {1, 2, 3} and edge set {(1, 2, 1), (1, 3, 1), (2, 3, 1)} where each triple (x, y, w) represents there is an edge with end vertices x, y, and weight w. Then the best and second best costs of spanning trees of this graph are both 2. For another graph with the same vertex set and edge set {(1, 2, 1), (1, 3, 2), (2, 3, 3)}, the best and second best costs of spanning trees of this graph are 3 and 4 respectively. Design an algorithm to solve this problem.