國立中央大學資訊工程學系 104 學年度第二學期博士班資格考試題紙

科目：　演 算 法　(Algorithms)　　　第一頁　共一頁(page 1 of 1)

1. (20%) A string is a sequence of symbols; for example, $X = <x_1,x_2,...,x_m>$ is a string of m symbols $x_1,x_2,...,x_m$. When we delete 0 or more symbols (not necessarily consecutive) from X, we get a subsequence of X. Write a dynamic programming algorithm to calculate the length of the longest common subsequence of $X=<x_1,x_2,...,x_m>$ and $Y=<y_1,y_2,...,y_n>$.

2. (20%) Write an algorithm to have the following input:
   a connected weighted digraph $G(V, E)$, w, x, and y, where V is the node set, E is the edge set, the weight of the edge (u, v) is stored in $w[u][v]$, and x and y are two nodes in V,
   and the following output:
   the shortest path from x to y.

3. (10%) How can we prove that a given problem H is NP-hard?

4. (25%)Consider a general version of coin changing problem: given a set of coins of different denominations with unlimited quantities, you are asked to make changes for K cents using these coins. Design an efficient algorithm to count the number of different ways of making changes. That is, if there are n kinds of coins of denominations $c_1, c_2, \ldots, c_n$, the problem is equivalent to asking you to design an algorithm to count the number of different solutions for the following integer linear programming problem: Given positive integers $c_1, c_2, \ldots, c_n$, and K, find non-negative integer solutions $(x_1, x_2, \ldots, x_n)$ such that $x_1c_1 + x_2c_2 + \ldots + x_nc_n = K$. For example, if $c_1 = 1$, $c_2 = 5$, $c_3 = 10$, and $c_4 = 25$, then there are four ways of making changes for 11 cents and there are 13 ways of making changes for 26 cents.

5. (25%) Given a list of n positive integers, $d_1, d_2, \ldots, d_n$, we want to efficiently determine whether there exists an undirected graph whose nodes have degrees precisely $d_1, d_2, \ldots, d_n$. This graph should not contain self-loops (edges with both endpoints equal to the same node) or multiple edges between the same pair of nodes. Design an algorithm to solve this problem.