

1. A string is a sequence of symbols; for example, $X = \langle x_1, x_2, \dots, x_m \rangle$ is a string of m symbols x_1, x_2, \dots, x_m . When we delete 0 or more symbols (not necessarily consecutive) from X , we get a subsequence of X . Write a dynamic programming algorithm to calculate the length of the longest common subsequence of $X = \langle x_1, x_2, \dots, x_m \rangle$ and $Y = \langle y_1, y_2, \dots, y_n \rangle$. (15%)
2. Show that the worst case lower bound of the sorting problem is $\Omega(n \log n)$. (15%)
3. What is the definition of the NP-hard problem? (10%) How to prove that a given problem X is NP-hard? (10%)
4. Ferries are used to transport cars across rivers. Typically, ferries are wide enough to support two lanes of cars throughout their length. The cars waiting to board the ferry form a single queue, and the operator directs each car in turn to drive onto the left or right lane of the ferry so as to balance the load. Each car in the queue has a different length, which the operator estimates by inspecting the queue. Based on this inspection, the operator decides which side of the ferry each car should board, and boards as many cars as possible from the queue, subject to the length limit of the ferry. The problem is to design an algorithm that will tell the operator which car to load on which side so as to maximize the number of cars loaded. (30%)
 - a) Give a decision version of this problem, and show that it is NP-complete.
 - b) Give a dynamic programming algorithm for solving this problem.
5. Given two lists of n nonnegative integers, r_1, r_2, \dots, r_n and c_1, c_2, \dots, c_n with $r_1 + r_2 + \dots + r_n = c_1 + c_2 + \dots + c_n$, we want to efficiently determine whether there exists a 0/1 $n \times n$ matrix whose n rows and n columns sums are the two given lists respectively. For example, the answer to the input $\langle (0,2), (1,1) \rangle$ is 'YES' since there is a matrix $\begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}$ satisfying the given condition and the answer to the input $\langle (0,2), (0,2) \rangle$ is 'NO'. Design an algorithm to solve this problem. (20%)