科目： **演 算 法** **(Algorithms)** 第一頁 共一頁（page 1 of 1）

1. (10%) By definition, show that $5n^3+2n-18$ is of $O(n^3)$ and $\Omega(n^3)$.

2. (20%) The shortest path problem is to find a path between two vertices (or nodes) in a weighted graph such that the sum of the weights of its constituent edges is minimized. Write the Floyd-Warshall all-pair shortest path algorithm for a weighted graph G=(V, E), where |V|=n, and G has positive or negative edge weights (but with no negative cycles), and the weight of the edge between node i and j is represented by the array element w[i][j]. You must show that the algorithm is of the $O(n^3)$ time complexity.

3. (20%) A vertex cover of a graph is a set of vertices such that each edge of the graph is incident to at least one vertex of the set. The problem of finding a minimum vertex cover is a classical optimization problem which is NP-hard. Write a polynomial-time 2-approximation algorithm to find a minimum vertex cover of a graph G=(V, E). You must show that your algorithm is a 2-approximation algorithm and is of the polynomial time complexity.

4. A *tree* is a connected undirected graph $G = (V, E)$ which contains no cycles. Many problems can be solved efficiently on trees. For example, the vertex cover problem is NP-hard for general graphs. However it can be solved in linear time on trees.
   (a) (10%) Show that the vertex cover problem is NP-hard for general graphs.
   (b) (20%) Design a linear time algorithm to solve this problem on trees.

5. (20%) A palindrome is a string that reads the same forward as it does backward. For example, the string ABABA is a palindrome. Design an algorithm to find the minimum number of operations required to make a given string to a palindrome. The operations are adding any character at any position, removing any character from any position, and replacing any character at any position with another character.