

1. (10%) By definition, show that $7n^4+2n^2+6n-5$ is of $\theta(n^4)$.
2. (20%) Write a prune-and-search algorithm of $O(n)$ time complexity to solve the selection problem: given a set S of n elements, find the k -th smallest element of S . You should show that your algorithm is of $O(n)$ time complexity.
3. (20%) Write a divide-and-conquer algorithm of $O(n \log n)$ time complexity to solve the closest pair problem: given n points on a plane, find a pair of points with the smallest distance between them. You should show your algorithm is of $O(n \log n)$ time complexity.
4. (30%) Given a weighted undirected graph $G(V, E, w)$, such that each edge e in E is assigned a non-negative weight $w(e)$, the problem Longest-Cycle is to find a longest cycle in G and the problem Shortest-Cycle is to find a shortest cycle in G . Decide which problem is polynomial-time solvable and which one is NP-hard. For the problem that is polynomial-time solvable, you should give an algorithm to solve it, and for the problem that is NP-hard, you should describe a known NP-hard problem and then show that the latter is polynomial time reducible to the former.
5. (20%) Show how to count the number of distinct substrings of a string T in time $O(n)$, where the length of T is n . Also show how to enumerate one copy of each distinct substring in time proportional to the length of all those strings.