國立中央大學資訊工程學系 101 學年度第一學期博士班資格考試題紙

1. (15%) Given $a_0, a_1, \ldots, a_{n-1}$, the Fast Fourier Transform (FFT) algorithm can compute $A_0, A_1, \ldots, A_{n-1}$ in O(n log n) time complexity by the divide-and-conquer strategy, where

$$A_j = \sum_{k=0}^{n-1} a_k e^{i2\pi jk/n}, 0 \le j \le n-1$$
$$= \sum_{k=0}^{n-1} a_k \omega^{kj}, \text{ where } \omega = e^{i2\pi/n}$$

. Show the basic concept of the FFT algorithm.

2. (15%) Given a set of n planar points $P_1, P_2, \ldots, P_n$, the Euclidean All Nearest Neighbor problem is to find the nearest neighbor of every P_i, $1 \le i \le n$. Show that the problem can be solved in linear time if the Voronoi diagram of the n points is given.

3. (20%) We have the following definitions and theorem related to NP-completeness.

Definition 1. Let A_1 and A_2 be tow problems. A_1 *reduces to* A_2 (written as $A_1 \propto A_2$) if and only if A_1 can be solved in polynomial time, by using a polynomial time algorithm which solves A_2.

Definition 2. A problem A is NP-complete if $A \in$ NP and every NP problem reduces to A.

Cook's Theorem. NP=P if and only if the satisfiability (SAT) problem is a P problem.

Let SAT $\propto$ B, B $\propto$ C, C $\propto$ D, and D$\in$NP for a given problem D. By the above definitions and theorem, show that D is NP-complete.
(Hint: You can first derive the reduction relationship of the SAT problem and every NP problem to complete the proof.)

4. Consider the problem to determine whether a given undirected graph with n vertices contains a cycle of length k as a subgraph. (a) (5%) Show that if k is a part of the input instance, then this problem is NP-complete. (b) (15%) However, if k is a constant, there is a naïve $O(n^k)$ time algorithm that solves this problem by checking all possible k vertices of the graph. Design a more efficient algorithm for this problem.

5. Let G be an undirected graph. Each edge e in G is assigned a probability p_e, $0 \le p_e \le 1$. The ***reliability*** of a path is the product of the probabilities of edges of the path. Design an efficient algorithm to find a path with maximal possible reliability. (15%)

6. Given a positive integer m, and an array $p[1..n]$ of n ($n \le m$) positive numbers, find a way to partition m into one or more positive integers $j_1, j_2, \ldots, j_k$ (i.e. $j_1 + j_2 + \ldots + j_k = m$) such that the sum $p[j_1] + p[j_1] + \ldots + p[j_k]$ is maximized over all possible partitions of m. (15%)