# A Decreasing *k*-Means Algorithm for the Disk Covering Tour Problem in Wireless Sensor Networks

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***Abstract*—This paper studies a Disk Covering Tour Problem (DCTP) for reducing the energy consumption of a mobile robot's movement to provide services for sensor nodes in a wireless sensor network (WSN). Given a set of locations of sensor nodes and a starting location of mobile robot, the DCTP is to find a minimum cost tour of a sequence of tour stops for the mobile robot to serve sensor nodes by keeping every sensor node within a specified distance of a tour stop. We propose an algorithm, called Decreasing *k*-means (D*k*-means), to find an approximate solution to the DCTP. The idea is to select a minimum number of disks or circles of a fixed radius to cover all sensor nodes, and then to find a minimum cost tour passing all disk centers. The simulation results show the proposed algorithm outperforms the related CSP (Covering Salesman Problem) algorithm and the QiF algorithm.**

***Keywords—disk covering tour problem; disk covering problem; covering salesman problem; wireless sensor network; k-means algorithm***

## I. Introduction

A wireless sensor network (WSN) consists of a large number of sensor nodes capable of sensing, computing, storing, and communicating data [1]. Each sensor node can sense physical phenomena, such as light, temperature, humidity, sound, and vibration, and can transmit sensed data through wireless transmission links. WSNs have wide applications like battlefield surveillance, environment monitoring, healthcare, and industrial sense. Recently, many WSN applications adopt wireless mobile robots to enhance their functionality. For example, some WSN applications use mobile robots for sensor deployment/relocation [16][12], search-and-rescue [9], wireless data collection [7][10], and wireless power charging of sensors [11][13].

This paper studies a Disk Covering Tour Problem (DCTP) for reducing the tour cost of a mobile robot in WSNs. Given a set of sensor nodes and a starting tour stop, the DCTP is to find a minimum cost tour for the mobile robot to leave from the starting tour stop, to pass through several tour stops for providing services, and to move back to the starting tour stop. The tour is under the *disk covering constraint* that every sensor node is *covered* by a disk that is centered at a tour stop and of a specified radius (or *cover range*). With the constraint, the mobile robot can provide services, such as wireless data collection and wireless power charging, for every sensor node. There are several ways to define the tour cost in the DCTP. In this paper, we assume the tour cost consists of the movement cost and the service cost. The movement cost corresponds to the energy consumption of the mobile robot in speeding up to leave tour stops, moving at a constant speed between two stops, and slowing down to reach stops. The service cost corresponds to the energy consumption of the mobile robot when it provides sensor nodes with services at tour stops. In general, the tour cost mainly depends on the number of tour stops and the total distance of the tour.

There are some problems proposed in the literature that is related to the DCTP. For example, some problems are to find the minimum number of tour stops for covering all or partial sensor nodes, such as the Disk Covering Problem (DCP), and the Disk Partial Covering Problem (DPCP) [15], which are NP-hard and can be solved by approximation algorithms, such as the greedy algorithm proposed in [15]. For another example, some problems are to find the shortest-distance (minimum-cost) tour visiting all or partial sensor nodes exactly once, such as the Travelling Salesman Problem (TSP) and the Covering Salesman Problem (CSP) [4], which are also NP-hard and can be solved nearly optimally by heuristic algorithms. For example, the Lin-Kernighan Heuristic (LKH) algorithm [6] and the CSP (or COVTOUR) algorithm [4] can return good solutions for the TSP and CSP, respectively. Furthermore, as will be shown later, the Qi-Ferry Problem (QiFP) is also related to the proposed DCTP and can be nearly optimally solved by the heuristic Qi-Ferry (QiF) algorithm [14].

This paper proposes a nearly optimal algorithm, called *Decreasing k*-means *(Dk*-means*)*, to solve the DCTP. The main idea of the D*k*-means algorithm is to first minimize the number of tour stops to meet the disk covering constraint, and then to minimize the total tour distance for keeping as low as possible the robot tour cost. To be more precise, the D*k*-means algorithm first divides all sensor nodes into a set of clusters by the *k*-means clustering method. It takes the cluster centers as tour stops for finding the nearly minimum number of tour stops to disk-cover all sensor nodes. Then, the algorithm finds a nearly shortest tour passing through all tour stops.

We simulate the proposed algorithm and compare it with related ones, namely the CSP algorithm and the QiF algorithm. The simulation results show that the tour returned by the proposed algorithm has a cost lower than those returned by the related algorithms.

The rest of this paper is organized as follows. Section II introduces some related work, and Section III describes the problem formulation and applications. The proposed algorithm and its performance simulations are introduced in Section IV and Section V, respectively. Finally, Section VI concludes this paper.

## II. REALTED WORK

In this section, we describe related problems, such as the DCP, DPCP, TSP, CSP, and Qi-Ferry problems [4, 14, 15]. We also introduce heuristic algorithms, namely the Lin-Kernighan Heuristic (LKH) algorithm [6], the greedy algorithms proposed in [2][15], and the Qi-Ferry algorithm [14], for solving some of the problems.

Xiao et al. in [15] studied the Disk Partial Covering Problem (DPCP), as described below. Given a set of $n$ nodes (or points) and a positive integer $p$, the DPCP is to find the minimum number $k$ of disks of a fixed radius $r$ to cover at least $p$ points. Note that when $p$ is set to be $n$, the DPCP becomes the Disk Covering Problem (DCP), which is to find the minimum number of fix-sized disks to cover all given points. Xiao et al. proposed a greedy algorithm, called the *greedy DPCP algorithm*, to solve the DPCP problem. The paper [2] adapted the greedy DPCP algorithm to solve the DCP for covering all sensor nodes in a WSN with the minimum number of disks by setting $p$ as $n$. Below, we called the DCP-version algorithm the *greedy DCP algorithm*.

The Travelling Salesman Problem (TSP) is a famous combination optimization problem, which is NP-hard, for finding a minimum cost tour visiting given *nodes* (or *tour stops*) exactly once. The TSP is NP-hard and can be solved by several heuristics. Lin-Kernighan Heuristic (LKH) is a famous heuristic to solve the TSP based on the concept of improving an existing tour iteration by iteration. The TSP has many variants, such as the Covering Salesman Problem (CSP), which is proposed by Current and Schilling in [4] and described as follows. Given a set $N$ of nodes and a covering distance $r$, the CSP problem is to identify the minimum cost tour visiting a subset of nodes in $N$, such that every node is either on the tour or is within $r$ distance of a node on the tour. The CSP has a variety of applications in the real world. For example, the paper [4] took the routing of a rural health care delivery team as an application of the CSP. The team is not necessary to directly visit each village; instead, it is more cost-efficient for the team to visit some selected villages, such that every village is selected or “near” one of the selected villages, where “near” is defined as “being within a predetermined distance of.” Current and Schilling in [4] also proposed a heuristic procedure, called the CSP (or COVTOUR) algorithm, for solving the CSP.

Li et al. in [8] studied the Qi-Ferry Problem (QiFP) about how to use an energy-constrained mobile charger, called the Qi-ferry, to wirelessly charge sensor nodes in a WSN so that a maximum number of sensors can be wirelessly charged and the Qi-Ferry can return to the starting point with residual energy. Li et al. used Particle Swarm Optimization (PSO) to develop a heuristic algorithm, called the QiF algorithm, to solve the QiFP. The QiF algorithm first finds a minimum energy tour covering all sensor nodes without considering Qi-ferry’s energy constraint by executing a greedy algorithm. Then the PSO-based heuristic is used iteratively to derive a tour covering as many as possible sensor nodes subject to the Qi-Ferry energy constraint.

## III. PROBLEM FORMULATION AND APPLICATIONS

In this section, we formulate the DCTP and show its possible applications. Below, we start with the DCTP formulation.

### A. Disk Covering Tour Problem (DCTP) Formulation

The Disk Covering Tour Problem (DCTP) is formulated as follows. Given a starting tour stop $s$, a node set $N$ of sensor nodes in the Euclidean space, and the disk radius $r$, the DCTP is to find a tour $T=<t_1=s, t_2, \ldots, t_w>$, which is an ordered list with $w$ tour stops, with a minimum cost $C(T)$ such that every node in $N$ is covered by at least a disk centered at a tour stop.

We define the cost $C(T)$ of the tour $T=<t_1=s, t_2, \ldots, t_w>$ as follows.

$$C(T) = \sum_{i=1}^{w}(M(t_i, t_{i(\mathrm{mod}\ w)+1}) + E(i)) \quad (1)$$

In Eq. (1), $M(t_i, t_{i(\mathrm{mod}\ w)+1})$ represents the movement cost for the mobile robot to move from tour stop $t_i$ to tour stop $t_{i(\mathrm{mod}\ w)+1}$. Note that we add (mod $w$) in the subscript for the case that the mobile robot moves from $t_w$ to $t_1=s$. It is easy to see that $\mathrm{M}(t_i, t_{i(\mathrm{mod}\ w)+1})$ is proportional to the distance between $t_i$ and $t_{i(\mathrm{mod}\ w)+1}$. Furthermore, $E(i)$ represents the cost for the mobile robot to provide the service at tour stop $t_i$, $1 \le i \le w$. For the sake of simplicity, we assume $E(i)$ is a fixed value $e$ for every tour stop $t_i$, $1 \le i \le w$. We thus have

$$C(T) = w \times e + \sum_{i=1}^{w} \mathrm{M}(t_i, t_{i(\mathrm{mod}\ w)+1}) \quad (2)$$

### B. Scenarios and Applications

The DCTP problem model fits into several real world scenarios. Fig. 1 shows the abstract of the scenario, in which a mobile robot starts the tour from a starting point at the upper left corner, visits every tour stop along the tour to cover all sensor nodes of a WSN, and finally goes back to the starting point. Note that the mobile robot does not need to go to the exact location of every sensor node. It just needs to stop at every tour stop to cover sensor nodes for providing them with services. Since the mobile robot is powered by batteries, it should spend as little as possible energy to finish the tour. As we have mentioned, the mobile robot is assumed to spend fixed energy at every tour stop to serve all sensor nodes in its service coverage no matter how many they are and where they are. So, we just consider the movement cost of the mobile robot.

The above-mentioned scenario abstract is applicable to several applications. Below are two of the examples.

1) Wireless Power Charger

For this application, the mobile robot carries a wireless charger and roves a WSN to wirelessly charge sensor nodes’ batteries so as to extend their lifetimes. When the mobile robot arrives near a sensor node and the wireless charger and the sensor node are within the wireless charging range, the wireless charger can charge the battery of the sensor node. In practice, the Qi-Ferry proposed in [8] is one of such applications.

2) Data Mule

A data mule is a mobile robot that physically carries a communication device with storage. It moves through remote locations to provide sensor nodes with the data exchange service. When the data mule arrives near a sensor node and the data mule and the sensor node are within the communication range, the data mule can collect data from the sensor node and/or can deliver data to the sensor node.

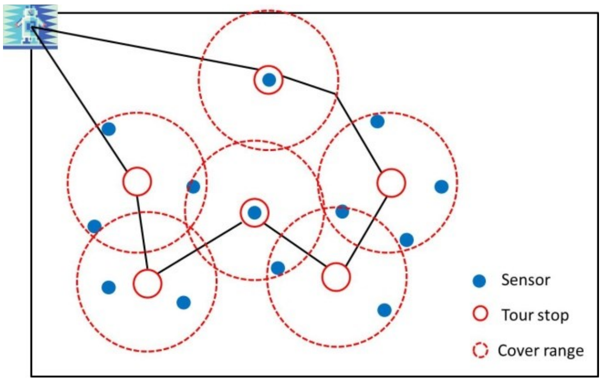


Fig. 1. A scenario of the Disk Covering Tour Problem (DCTP).

## IV. DECREASING *K*-MEANS ALGORITHM

In this section, we elaborate the proposed Decreasing *k*-means (D*k*-means) algorithm to solve the DCTP. The main notion of the D*k*-means algorithm is to first minimize the number of tour stops, and then to minimize the distance of the tour visiting all tour stops with the constraint that every sensor node in given nodes set $N$ is within the distance $r$ (the disk radius or the service coverage range) of a tour stop.

The D*k*-means algorithm has three phases. In the first phase, it gets $k$ tour stops by executing the greedy DCP algorithm [2][15] for the purpose of obtaining the initial value of $k$. The second phase has many iterations, say $m$ iterations. At each iteration, the D*k*-means algorithm finds a set of $k$ cluster centers by executing the *k*-means clustering algorithm. It then adjusts the position of each cluster center to be the center of the smallest circle covering all nodes in the cluster by executing the well-known polynomial-time Smallest Enclosing Circle (SEC) algorithm. If any circle associated with each cluster has a radius larger than the pre-specified disk radius $r$, then the algorithm continues the next iteration. Otherwise, the algorithm tries to find a tour by executing the LKH algorithm. The algorithm then calculates the tour cost $C'$ of $T'$. If $C'\geq C$, then the next iteration continues. If $C'<C$, then $C$ is set as $C'$, $T$ is set as $T'$, $k$ is decreased by 1, the number of remaining iterations in the second phase is reset as $m$, and the next iteration continues, where $C$ and $T$ are used to store the minimum tour cost ever found and the associated tour, respectively.

The second phase of the D*k*-means algorithm stops after $m$ iterations or stops when $k$<1. Afterwards, the algorithm goes to the third phase and returns the best tour $T$ and its cost $C$ found in the second phase.

```
Algorithm: Dk-means for DCTP
Input: N, s, r, m
//N: node set; s: starting stop; r: radius; m:max iterations
Output:  The tour T with the near minimum cost C
C=∞; T=null;
Execute the greedy DPCP (Disk Partial Covering Problem) algorithm to get the initial value of k
for (i=m; i≥1 and k≥1; i--)
    Execute the k-means algorithm to get k centers of k clusters
    Adjust the locations of the k centers by executing the SEC (Smallest Enclosing Circle) algorithm
    if (there exists one node not within distance r of any of the k centers) then Continue the next for-loop iteration
    else
        Execute the LKH algorithm to get the tour T′ with cost C′
        if (C′<C) then
            C=C′; T=T′; k=k-1; i=m;
        else Continue the next for-loop iteration
return T and C
```

Fig. 2. The Pseudo Code of the D*k*-means algorithm.

## V. SIMULATIONS

### A. *Simulation Setting*

We develop a simulator on the MATLAB platform to evaluate the performance of the proposed D*k*-means algorithm. The simulated scenario is the random deployment of 30, 60, …, and 120 sensor nodes in a 100 m by 100 m area. The simulation is executed for 10,000 iterations in the second phase of the algorithm. The disk radius $r$ is 10 m, and the starting tour stop is at the upper left corner. Initially, the sensor nodes within the $r$ distance of the starting tour stop will be served.

The robot movement energy consumption model is based on the experimental results proposed in [14]. The speed of the motor is 3600 rad/sec, whose energy consumption results are shown in Fig. 3, where the X-axis is the moving distance of the mobile robot and Y-axis is the energy consumption of moving distance per meter. As the result shown in the Fig. 3, the energy consumption per meter decreases with the moving distance and approaches constant when the moving distance is more than 2 m. This is because much energy is spent on speed acceleration (i.e., speeding up and slowing down) when the moving distance is less than 2 m. When the moving distance is larger than 2 m, the distance contains a large portion of constant speed, which consumes little energy.

### B. *Performance Comparsions*

As shown in Fig. 4, the number of tour stops of our proposed D*k*-means algorithm is smaller than the greedy DCP algorithm [2][15] (denoted by "Greedy" in Fig. 4), the CSP algorithm [4] and the QiF algorithm [8] (denoted by Qi-Ferry in Fig. 4). Note that the tour stops of both the CSP algorithm and the QiF algorithm need to be located at given nodes, leading to larger numbers of tour stops. On the other hand, the D*k*-means algorithm and the greedy DCP algorithm can select tour stops at any location, leading to smaller number of tour stops. The D*k*-means algorithm improves the greedy DCP algorithm by adjusting cluster center positions by executing the SEC algorithm to achieve better performance.

As shown in Fig. 5, the total tour length of the D*k*-means algorithm is less than the CSP algorithm. However, it is slightly more than that of the QiF algorithm in the cases of 30 nodes and 40 nodes. This is because that the PSO heuristic used by the QiF algorithm has good performance in finding solutions in these cases. In the cases of 90 nodes and 120 nodes, the performance of the QiF algorithm is not so good. This is because the PSO heuristic has higher probability to fall into local optima when given a large number of nodes. The CSP has the longest distance, since its selection mechanism is based on the given nodes. Note that the greedy DCP algorithm is not included in the comparison, since it does not generate tours.

Fig. 6 shows the comparisons of the tour costs (energy consumptions) for the D*k*-means algorithm, the CSP algorithm and the QiF algorithm. In the comparisons, the tour cost is calculated according to Eq. (1) and Eq. (2), in which the movement cost is based on the experimental results proposed in [14]. As shown in Fig. 6, among the three algorithms compared, the D*k*-means algorithm is almost the best for all cases. It has slightly higher tour costs than the QiF algorithm only in the case of 30 nodes. However, the D*k*-means algorithm outperforms both the QiF algorithm and the CSP algorithm as the number of nodes increased. In summary, the D*k*-means algorithm has better performance for the case of the low node density (i.e., the small number of nodes in a fixed-sized area).

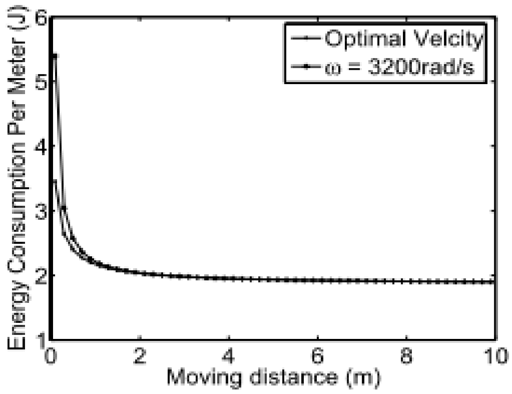


Fig. 3. The energy consumption of a mobile device moving over a moving distance (adapted from [14]).

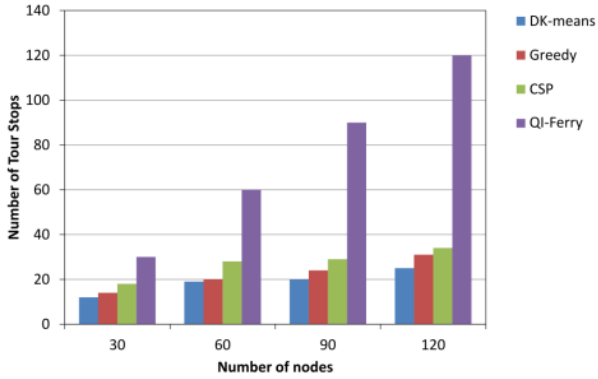


Fig. 4. The comprasions of the number of tour stops for differnet algoritms.

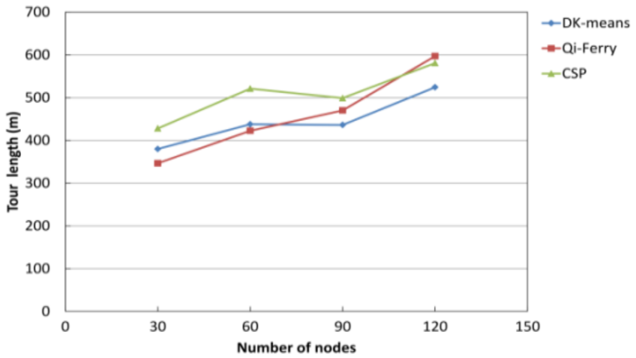


Fig. 5. The comprasions of the tour distance (tour length) for different algorithms.

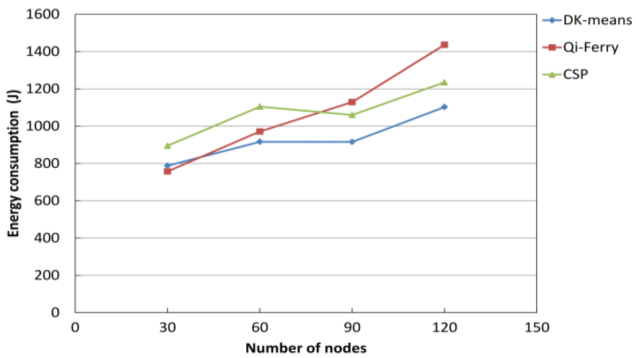


Fig. 6. The comprasions of tour cost (energy consumption) for different algorithms

## VI. CONCLUSIONS

In this paper, we have discussed the DCTP and proposed the D*k*-means algorithm to solve it for the application of serving all sensor nodes in a WSN by a mobile robot moving along a near minimum cost tour. The main notion of the D*k*-means algorithm is to first minimize the number of tour stops in the tour, and then to minimize the distance of the tour visiting all tour stops with the constraint that every sensor node is within the specified distance of a tour stop. We have performed a series of simulations to evaluate the performance of the D*k*-means algorithm and related methods, namely the greedy DCP algorithm [2][15], the CSP algorithm [4] and the QiF algorithm [8]. The simulation results show the D*k*-means algorithm outperforms the others for almost all cases.

In the future, we plan to improve our work in finding smaller numbers of tour stops by special heuristic algorithms, such as the genetic algorithm, and the ant colony optimization algorithm. Moreover, we also plan to improve the tour cost calculation by considering more subtitle factors.

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