# Optimizing Sink-Connected Barrier Coverage in Wireless Sensor Networks

Yung-Liang Lai[1] and Jehn-Ruey Jiang[2*]

[1]Department of Computer Science and Information Engineering

Taoyuan Innovation Institute of Technology, Jhongli City, Taoyuan, 32091, Taiwan, R.O.C.

Email: yungliang.lai@gmail.com

[2]Department of Computer Science and Information Engineering

National Central University, Jhongli City, Taoyuan, 32001, Taiwan, R.O.C.

Email: jrjiang@csie.ncu.edu.tw

* Corresponding author

**Reference: to this paper should be made as follows: Lai, Y-L. and Jiang, J-R. (xxxx) 'Optimizing Sink-Connected Barrier Coverage in Wireless Sensor Networks', Int. J. Ad Hoc and Ubiquitous Computing, Vol. x, No. x, pp.xxx–xxx.**

**Biographical notes: Yung-Liang Lai received his Ph. D. degree in Computer Science and Information Engineering from National Central University, Taiwan. His research interests include mobile computing, wireless networks, wireless sensor networks, and optimization theory.**

**Biographical notes: Jehn-Ruey Jiang received his PhD in Computer Science from National Tsing-Hua University, Taiwan, in 1995. He is currently with the Department of Computer Science and Information Engineering, National Central University, Taiwan, and co-leads Adaptive Computing and Networking (ACN) Laboratory. He was Guest Editor of International Journal of Ad Hoc and Ubiquitous Computing (IJAHUC) and Journal of Information Science and Engineering (JISE). His research interests include internet of things, 4G networks, wireless sensor networks and network virtualization.**

## Abstract

This paper proposes an algorithm, named the optimal node selection algorithm (ONSA), to solve the sink-connected barrier coverage optimization problem, which is concerned with how to select randomly deployed sensor nodes of a wireless sensor network (WSN) to reach two optimization goals: (1) to maximize the

degree of barrier coverage with the minimum number of detecting nodes, and (2) to make the detecting nodes sink-connected with the minimum number of forwarding nodes. The detecting nodes are those for detecting intruders crossing a belt-shaped area of interest. On detecting intruders, they send intruding event notifications to one of the sink nodes with the help of the forwarding nodes to relay the notifications. We prove the optimality of ONSA, analyze its time complexity, perform simulations for it, and compare the simulation results with those of a related algorithm to show ONSA's advantages.

Keywords: Wireless sensor networks; Barrier coverage; Maximum flow minimum cost algorithm; Sink connectivity; Energy efficiency

## 1. INTRODUCTION

A wireless sensor network (WSN) consists of a large number of sensor nodes with the capabilities of sensing, computing, storing, and communicating data. Each sensor node can sense physical phenomena, such as light, temperature, sound, vibration, or electromagnetic field strength, and can transmit sensed data to one or more sink nodes through multiple-hop transmission links. WSNs are self-organizing in the sense that they can form without human intervention, adapt to node failure and degradation, and react to task changes. They have wide applications like battlefield surveillance, environment monitoring, industrial sense, and so on. Some recent research uses the WSN to establish a virtual barrier of sensor nodes for detecting intruders crossing a protected area boundary, such as coastlines, national borders [Kumar et al. 2007], and battlefield perimeters [Saipulla et al. 2009].

The barrier coverage problem deals with how to deploy WSN sensor nodes to form barrier coverage for detecting intruders crossing a belt-shaped area of interest, which is defined as an area between two parallel curves (e.g., a rectangular area). As illustrated in Fig. 1, the monitored rectangular area is deployed with some sensor nodes to detect any intruder that penetrates the area by first passing through the outer side and then passing through the inner side of the area. Several studies [Balister et al. 2007] [Cardei and Wu 2006] address the problem. Some of them try to measure the quality of barrier coverage and/or to design schemes achieving high-quality barrier-coverage in WSNs. In general, the quality of barrier coverage is measured by the degree. A WSN is said to form *k*-degree barrier coverage (or *k*-barrier coverage, for short) if any intruder crossing the barrier is to be detected by at least *k* sensor nodes. To take the WSN in Fig. 1 as an example again, it forms 2-barrier coverage and its degree of barrier coverage is 2. This is because any intruder will be detected by at least two different sensor nodes when the intruder crosses the WSN from the outer side to the opposite inner side.

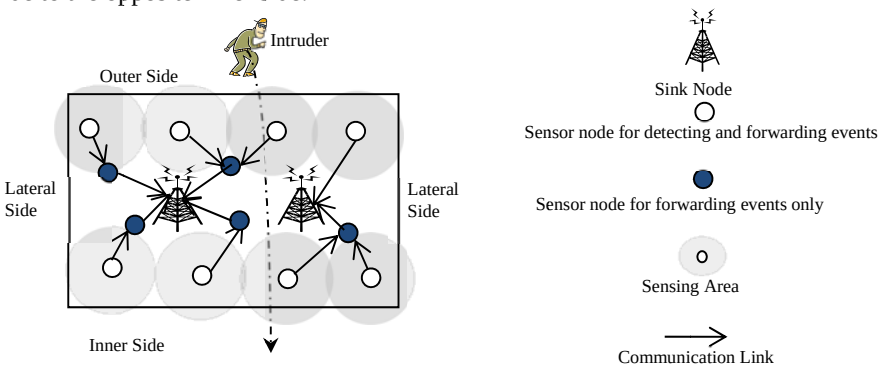



Fig. 1. An example of sink-connected 2-barrier coverage

To the best of our knowledge, no earlier research addresses the barrier coverage problem with both the considerations that the sensor nodes should be connected to the sink node and that the

number of the sensor nodes is minimized. In this paper, we take both considerations into account and propose an algorithm to solve the *sink-connected barrier coverage optimization problem*, dealing with how to select sensor nodes of a randomly deployed WSN to reach the following two goals:

Goal 1: Maximizing the degree of barrier coverage with the minimum number of detecting nodes

Goal 2: Minimizing the number of forwarding nodes that make detecting nodes sink-connected

Randomly deployed nodes can be selected to be detecting nodes or forwarding nodes. The former is selected to be active for detecting intruders and sending/forwarding intruding event notifications towards the sink nodes; and the latter, only for forwarding the notifications. It is noted that unselected nodes can remain inactive to save energy; however, when WSN topology changes due to node/link failures, inactive nodes can become active to make WSNs survive the failures, as discussed later. The first goal is to maximize the degree of WSN barrier coverage, while minimizing the number of detecting nodes. The second goal is to make detecting nodes sink-connected (i.e., to make sure that every detecting node can find a path to send intruding event notifications to a sink node) by adding a minimum number of forwarding nodes. Since the two goals minimize the number of active nodes (i.e., the detecting nodes and the forwarding nodes), the totally energy consumption is reduced.

The proposed algorithm to solve the sink-connected barrier coverage optimization problem, which is called the optimal node selection algorithm (ONSA), is based on the maximum flow algorithm [Cormen et al. 2001] and minimum cost algorithm [Orlin and Ahuja 1992]. The paper [Kumar et al. 2007] also uses the maximum flow algorithm to determine whether a belt-shaped area is *k*-barrier covered or not. However, it does not address the problem about how to achieve the *k*-barrier coverage with the minimum number of nodes. It does not consider the sink-connected property, either. In practice, no research considers the sink-connected property as defined in this paper. The proposed algorithm is to achieve the maximum degree of barrier coverage with the minimum number of nodes and to attain the sink-connected property with the minimum number of nodes for a WSN with randomly deployed nodes. We prove the optimality of ONSA, perform simulation experiments for ONSA, and compare the simulation results with those of the related algorithm, the global determination algorithm (GDA), proposed in [Kumar et al. 2007].

The remainder of this paper is organized as follows. In Section 2, we introduce some related work. In Section 3, we present the network model and problem definition. We describe the proposed algorithm, prove its optimality, and analyze its time complexity in Section 4. Some simulation results are reported in Section 5. Finally, conclusion is drawn in Section 6.

## 2. RELATED WORK

Many studies [Balister et al. 2007] [Cardei and Wu 2006] addressed coverage problems for the WSN, which are classified into three classes: the point coverage problem, the area coverage problem and the barrier coverage problem [Saipulla 2011]. In the point coverage problem, the subject to be covered is a set of discrete points used to model some physical targets, such as missile launchers, at specific locations. The area coverage problem discusses how to cover every point in a specified area of interest, and the barrier coverage problem investigates how to cover any path penetrating a specified belt-shaped region. Below, we describe some studies related to the barrier coverage problem, which is the focus of the paper.

The notion of barrier coverage was first introduced by Gage in [Gage 1992] for sensor-based battlefield boundary surveillance to minimize the probability of undetected enemies penetrating through the boundary. In [Liu and Towsley 2004], Liu and Towsley defined detectability to be the probability that a WSN is able to detect intruders moving through a belt-shaped region of interest. They investigated detectability in many aspects and showed that if the sensor node density is below a threshold, an intruder can almost surely find a path to cross the region without being detected. Wang and Cao in [Wang and Cao 2011] studied how to construct barrier coverage to

monitor moving objects in camera sensor networks with consideration of the viewing direction and the sector-shaped coverage area of camera sensors.

Kumar et al. [Kumar et al. 2007] defined the notion of *k*-barrier coverage for precisely representing a WSN's ability of intruder detection. A WSN is said to have the *k*-barrier coverage property if any intruder crossing the virtual barrier of a monitored area is detected successful by at least *k* sensor nodes. The authors developed theorems and proposed a centralized scheme using the maximum flow algorithm to determine whether a belt-shaped area is *k*-barrier covered or not. Besides, they showed that an individual sensor node cannot locally decide whether a network can form barrier coverage due to the lack of the global information. Unlike the algorithm in [Kumar et al. 2007] that returns either true or false (0 or 1) for measuring the quality of barrier coverage, the method proposed by Chen et al. in [Chen et al. 2008] returns a non-binary value for the measurement. They also proposed a method to identify local regions whose qualities do not reach the desired degree of coverage.

Chen et al. [Chen et al. 2007] proposed a localized algorithm that guarantees the detection of intruders whose trajectory is confined to a slice of a belt-shaped area. Saipulla et al. in [Saipulla et al. 2009] studied the barrier coverage of WSNs with line-based deployment, in which sensors are deployed along a line (e.g., sensors are dropped from an aircraft along a given path). Balister et al. [Balister 2007] estimated the reliable node density that achieves barrier coverage with *s*-*t* connectivity in a thin strip of boundary with finite length, where *s*-*t* connectivity means that a connected path exists between the two far ends of the thin strip.

## 3. NETWORK MODEL AND PROBLEM FORMULATION

In this section, we first describe the network model and then formulate the sink-connected barrier coverage optimization problem to be solved in this paper.

### 3.1 Network Model

Consider a WSN consisting of many sensor nodes and few sink nodes, in which sensor nodes are to form a virtual sensor barrier for monitoring a belt-shaped area to detect and send intruding events to one of the sink nodes. The sensor nodes are assumed to be randomly deployed; for example, they can be dropped from an aircraft as described in [Saipulla et al. 2009]. Each sensor node is equipped with a sensing module with a fixed sensing range to sense intruders and a communication module with a fixed communication range to communicate with other sensor nodes or sink nodes. Note that no assumption is put for the relationship of the sensing range and the communication range.

Initially, a sink node broadcasts a command to make every sensor node perform a bootstrapping task to pin point its location, discover the nodes within its communication range, and report its information, such as the identification and the location, to one of the sink nodes. The sink nodes are more powerful than sensor nodes. They have more energy, memory, computing power and communication capacity. They can communicate with each other and with sensor nodes; they can also communication with the backend system, which is assumed to have unlimited power supply and enormous computing power to gather all WSN nodes' information and perform the optimization computation.

Let *Vs* and *Vk* denote the set of sensor nodes and the set of sink nodes, respectively. Below, we define a coverage graph *Gc* to represent the sensing area coverage relationships of nodes. Moreover, we define a transmission graph *Gt* to represent the nodes' wireless transmission reachability relationships.

(1) Coverage Graph

A coverage graph $Gc(Vs \cup \{S,T\}, Ec)$ is a directed graph, in which *Vs* is the sensor node set, *Ec* is the edge set, and *S* is a virtual source node and *T* is a virtual target node. The edge set *Ec* represents the sensing area coverage overlap relationships. For two nodes $N_i$ and $N_j$ in *Vs*, edges $(N_i, N_j)$ and $(N_j, N_i)$ exist in *Ec* if $N_i$'s coverage and $N_j$'s coverage have overlap. As shown in Fig. 2,

the belt-shaped area of interest has the outer side, inner side and lateral sides. Intruders are supposed to cross the belt-shaped area from outer side to inner side. Virtual nodes *S* and *T* are associated with the lateral sides; to be more precise, an edge ($N_i$, *S*) or ($N_i$, *T*) exists in *Ec* if $N_i$'s sensing area overlap either lateral side. Fig. 2 shows the coverage graph *Gc* of the WSN with 8 sensor nodes $N_1$,..,$N_8$, which are represented by solid circles. Note that the gray shades around the solid circles represent the sensing areas of sensor nodes.

Now, we can define the traversable paths in *Gc*. A traversal path of a coverage graph *Gc*(*Vs*∪{*S*,*T*}, *Ec*) is defined to be a path starting from *S*, going along edges in *Ec* through nodes in *Vs*, and stopping at *T*. Note that a coverage graph is similar to a flow network [Ahuja et al. 1993] and a traversable path is similar to a flow in the network. In the flowing context, the terms "traversable path" and "flow" will be used alternatively. The coverage graph and its traversal paths are very useful for measuring the degree of barrier coverage. By the theorems developed in [Kumar et al. 2007], a WSN forms *k*-barrier coverage if and only if *k* node-disjoint traversable paths exist in the coverage graph of the WSN. In the WSN of Fig. 2, there are two node-disjoint traversable paths (e.g., *S*-$N_1$-$N_2$-$N_3$-$N_4$-*T* and *S*-$N_5$-$N_6$-$N_7$-$N_8$-*T*) in the WSN coverage graph, so the WSN forms 2-barrier coverage.

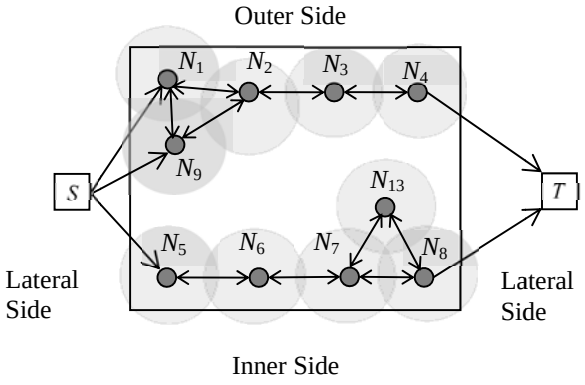



Fig. 2. A WSN coverage graph with 2 node-disjoint traversable paths

(2) Transmission Graph

A transmission graph *Gt*(*Vs*∪*Vk*, *Et*) is a directed graph, where *Vs* is the sensor node set, *Vk* is the sink node set, and *Et* is an edge set to represent transmission relationships. For two nodes $N_i$ and $N_j$ in *Vs*, an edge ($N_i$, $N_j$) exists in *Et* if the node $N_i$ can successfully transmit data (or events) to node $N_j$ over a direct wireless link. Based on the transmission graph *Gt* of a WSN, we can define the sink-connected property for a set of sensor nodes as follows. For the WSN with the transmission graph *Gt*(*Vs*∪*Vk*, *Et*), a set *Q* (*Q*⊆*Vs*) of sensor nodes is sink-connected, if for each node in *Q*, there is a path going through only nodes in *Q* to reach a node in *Vk*. For example, for the WSN in Fig. 3 consisting of 14 sensor nodes $N_1$,…,$N_{14}$ and 2 sink nodes $K_1$ and $K_2$, the node sets {$N_4$}, {$N_{11}$}, {$N_1$, $N_9$}, {$N_2$, $N_3$, $N_{11}$}, {$N_4$, $N_7$, $N_8$, $N_{13}$}, {$N_9$,…,$N_{13}$} and {$N_1$,...,$N_{13}$} all satisfy the sink-connected property. However, the node sets {$N_1$}, {$N_6$, $N_{11}$} and {$N_1$,…,$N_8$} do not satisfy the sink-connected property.

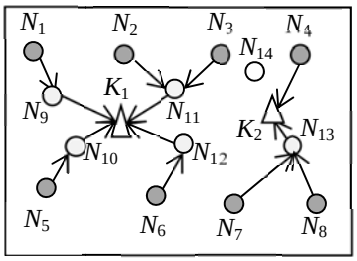



Fig. 3. A WSN transmission graph with partial edges for illustrating the sink-connected property

### 3.2 Sink-Connected Barrier Coverage Optimization Problem

The objective of the sink-connected barrier coverage optimization problem is to maximize the degree of barrier coverage of a WSN by selecting the minimum number of nodes, while keeping the selected nodes sink-connected. Below, we formally define the problem.

Given a WSN with the coverage graph $Gc(Vs \cup \{S,T\}, Ec)$ and the transmission graph $Gt(Vs \cup Vk, Et)$, the sink-connected barrier coverage optimization problem is to achieve the following two goals.

Objective 1: To find a minimum sensor node set *Vd* such that the number of node-disjoint traversable paths of *Vd* is maximized

Objective 2: To find a minimum forwarding node set *Vf* such that $(Vd \cap Vf=\varnothing)$ and $(Vd \cup Vf)$ satisfies the sink-connected property

According to the above definition, a solution to the sink-connected barrier coverage optimization problem will return two node sets *Vd* and *Vf*. The nodes in *Vd* are regarded as detecting nodes to detect intruding events; and the nodes in *Vf*, forwarding nodes to forward event notifications to one of the sink node. Since the detecting nodes remain active, they can certainly help forward the event notifications sent by other detecting nodes. The solution is optimal in the sense that the degree of barrier coverage is maximized, while the number of detecting nodes and the number of forwarding nodes are both minimized. The solution is also practical in the sense that the detecting nodes are sink-connected with the help of forwarding nodes.

## 4. THE OPTIMAL NODE SELECTION ALGORITHM (ONSA)

### 4.1 Algorithm Description

In this section, we propose an algorithm, called the optimal node selection algorithm (ONSA), to solve the sink-connected barrier coverage optimization problem. Given the sensor nodes *Vs*, sink nodes *Vk*, coverage relationship *Ec*, and transmission relationship *Et*, ONSA can find the detecting node set *Vd* and the forwarding node set *Vf*. ONSA has four main tasks. The first task is to construct the coverage graph *Gc* and then perform the node-disjoint transformation to generate the graph *Gc** such that *Gc** is a flow network [Cormen et al. 2001]. The second task is to find a flow plan in Gc* by executing the minimum cost maximum flow algorithm [Cormen et al. 2001] [Orlin and Ahuja 1992]. The third task is to construct the transmission graph *Gt* and then perform the node-edge transformation based on the flow plan returned in the second step to generate the graph *Gt** that is a flow network. The fourth task is to find the final flow plan in *Gt** by again executing the maximum flow minimum cost algorithm. The nodes appearing in the final flow plan

will be activated for constructing sink-connected barrier coverage with the maximum degree. The pseudo code of ONSA is shown in Fig. 4 and explained below.

In step 1, ONSA constructs a coverage graph *Gc* with a virtual node *S* and a virtual node *T*. The edges incident to the sink node are associated with Capacity 1 and Cost 0, and all other edges are associated with Capacity 1 and Cost 1.

In step 2, ONSA executes the node-disjoint transformation to covert *Gc* into *Gc**. As shown in Fig. 5(a), the node-disjoint transformation changes a node *X* with multiple inbound flows and multiple outbound flows into a pair of virtual nodes *X'* and *X''* which has an edge going from *X'* to *X''* associated with Capacity=1 and Cost=0. The purpose of the transformation is to make the generated flow plan in *Gc** node-disjoint.

In step 3, ONSA executes the maximum flow minimum cost algorithm on *Gc** to decide the flow plan *Fc*. The maximum flow minimum cost algorithm has two procedures. The first procedure is to find the maximum flow by executing the Edmonds-Karp algorithm [Cormen 2001]. The second procedure is to find the minimum cost flow by executing the Orlin-Ahuja algorithm [Orlin and Ahuja 1992]. The readers are referred to [Cormen 2001] and [Orlin and Ahuja 1992] for the procedure details. In this step, the nodes selected in *Fc* are included in the node set *Vd*, the set of detecting nodes. Since the flow in Fc is maximized, the number of node-disjoint traversable paths in *Vd* is also maximized. Moreover, since the cost of *Fc* is minimized, the number of nodes in *Vd* is also minimized.

In step 4, ONSA constructs a transmission graph $Gt(Vs \cup Vk, Et)$ and adds a virtual source node *S* and a virtual target node *T* into *Gt*.

In step 5, ONSA inserts a virtual source node *S* into *Gt* and adds an edge between the node *S* and every detecting node in *Vd*. Each newly added edge is associated with Capacity=1 and Cost=0. ONSA also inserts a virtual target node *T* into *Gt* and adds an edge between the node *T* and every sink node in *Vk*. Each newly added edge is associated with Capacity=$\infty$ and Cost=0. The settings of Capacity and Cost are to guarantee that every detecting node in *Vd* has a flow going to one of the sink nodes.

In step 6, ONSA executes the node-edge transformation to covert *Gt* into *Gt**. As shown in Fig. 5(b), the node-edge transformation changes each node (excluding *S* and *T*) into two virtual nodes with one edge of Capacity=$\infty$ and Cost=1. The purpose of the transformation is to make the obtained flow plan in *Gt** have the minimum number of nodes.

In step 7, ONSA executes the maximum flow minimum cost algorithm on *Gt** to decide the flow plan *Ft*. In this step, the nodes selected in *Ft* are included in the node set *Vm*. Since *Ft* has the minimum cost, the number of nodes in *Vm* is minimized.

In step 8, ONSA returns *Vd* as the set of detecting nodes, and returns $Vf = Vm - Vd$ as the set of forwarding nodes.

---

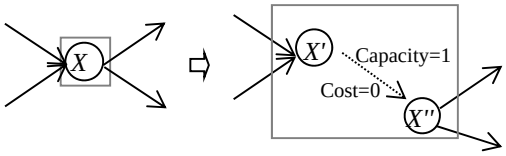
Optimal Node Selection Algorithm (ONSA)

---

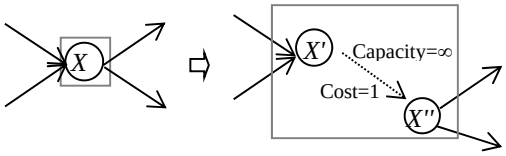
Input: *Vs*, *Vk*, *Ec*, *Et*
Output: *Vd* and *Vf*
Step 1: Construct a coverage graph $Gc(Vs \cup \{S,T\}, Ec)$, where *S* and *T* are virtual nodes, and associate all edges incident to *T* with Capacity=1 and Cost=0, and all other edges with Capacity=1 and Cost=1.
Step 2: Execute the node-disjoint transformation to convert *Gc* into *Gc**.
Step 3: Execute the maximum flow minimum cost algorithm to obtain the minimum cost flow plan *Fc* on *Gc**. Let node set *Vd*, $Vd \subseteq Vs$, be the set of nodes associated with *Fc*.

Step 4: Construct a transmission graph $Gt(Vs \cup Vk, Et)$, where each edge is with Capacity=1 and Cost=0. Add a virtual source node $S$ and a virtual target node $T$ into $Gt$.
Step 5: For each node in $Vd$ on graph $Gt$, add an edge going from $S$ to it with Capacity=1 and Cost=0. For each sink node in $Vk$, add an edge going from it to $T$ with Capacity=$\infty$ and Cost=0.
Step 6: Execute the node-edge transformation to convert $Gt$ into $Gt$*.
Step 7: Execute the maximum flow minimum cost algorithm to obtain the minimum cost flow plan $Ft$ on $Gt$*. Let $Vm$, $Vm \subseteq Vs$, be the set of the nodes associated with $Ft$.
Step 8: Return $Vd$ as the set of detecting nodes. Let $Vf$ be ($Vm - Vd$), and return $Vf$ as the set of forwarding nodes.

Fig. 4. The pseudo code of the ONSA algorithm




(a) Node-Disjoint Transformation




(b) Node-Edge Transformation

Fig. 5. Two transformations of ONSA

Below, we take the WSN in Fig. 2 as an example to illustrate the execution of ONSA. In step 1, a coverage graph $Gc$ will be constructed. After step 2, the nodes with multiple inbound flows and multiple outbound flows are transformed by the node-disjoint transformation. The transformation results are shown in Fig. 6. In step 3, a flow plan is obtained by the maximum flow minimum cost algorithm. In this example, the maximum number of flows is two and the set of nodes $Vd$ associated with $Fc$ is $\{N_1,\ldots,N_8\}$.

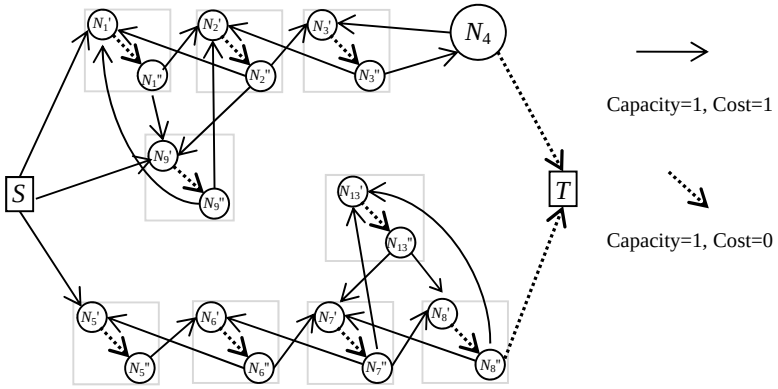



Fig. 6. An example of the node-disjoint transformation on a coverage graph

Now, we take the WSN in Fig. 3, which is extended from that in Fig. 2, as another example to illustrate the execution of ONSA. Note that we assume nodes $N_1,\ldots,N_8$ have been selected as detecting nodes in *Vd*. In step 4, the graph transmission graph *Gt* is constructed and virtual nodes *S* and *T* are added into *Gt*. In step 5, a new edge is added between the virtual source node *S* and every node in *Vd*. Moreover, a new edge is added between every sink node in *Vk* and the virtual target node *T*.

In step 6, the node-edge transformation is performed to convert *Gt* into *Gt**, as shown in Fig. 7. In step 7, the maximum flow minimum cost algorithm is executed to obtain *Vm*. In this example, *Vm* is $\{N_1,\ldots,N_{13}\}$, which is a set containing the nodes selected in *Ft*.

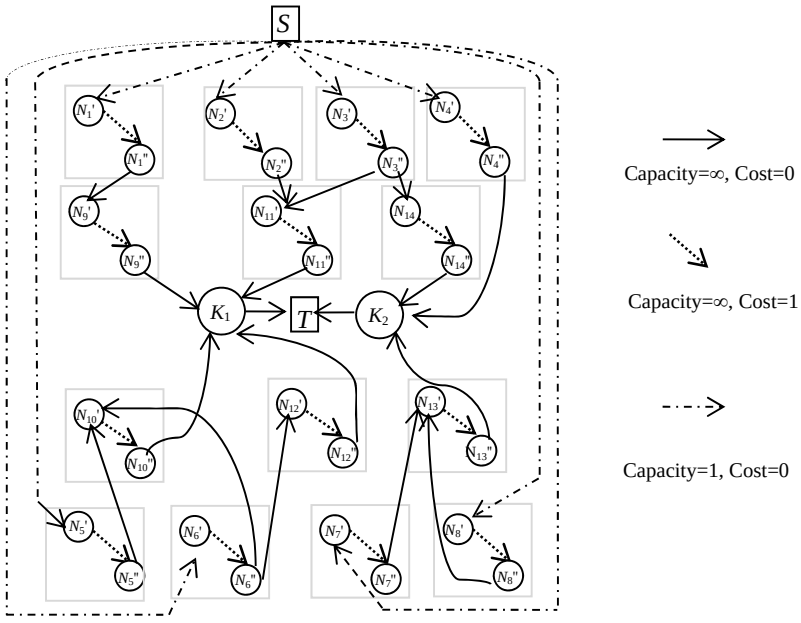



Fig. 7. An example of the node-edge transformation on a transmission graph

In step 8, $Vd=\{N_1,\ldots,N_8\}$ and $Vf=\{N_9,\ldots,N_{13}\}$ are returned by ONSA, where *Vd* is the set of detecting nodes to form 2-barrier coverage for detecting intruding events and *Vf* is the set of forwarding nodes to forward events notifications sent by detecting nodes to one of the sink nodes (i.e., either $K_1$ or $K_2$). Fig. 8 shows the execution results returned by ONSA.

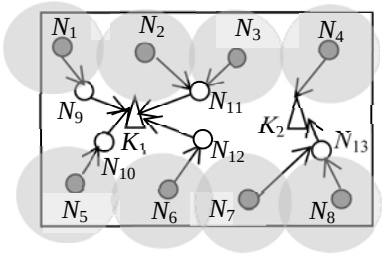



Fig. 8. The execution result of ONSA, where $N_1,\ldots,N_8$ are selected as detecting nodes and $N_9,\ldots,N_{13}$ are selected as forwarding nodes to forward event notifications to one of the sink nodes $K_1$ and $K_2$

The ONSA algorithm does not put any assumption about the relationship of the nodes' sensing range and communication range. For the special case that the communication range is at least twice as large as the sensing range, the nodes selected for maintaining barrier coverage (i.e. the detecting nodes in *Vd*) are connected but are not necessarily connected to sink nodes. The special case still needs the ONSA algorithm to find out the forwarding node set $Vf=Vm-Vd$ to make every detecting node connected to a sink node via intermediate nodes in $(Vd \cup Vf)$. Note that it is possible for a node to be in *Vd* but not in *Vm*, which means that the node just plays the role of the detecting node but does not need to forward event notifications for other detecting nodes.

### 4.2 Optimality Proofs

Now, we show by the following two theorems that the node sets *Vd* and *Vf* returned by ONSA achieve Goal 1 and Goal 2 of the sink-connected barrier coverage optimization problem.

**Theorem 1**. Let *Gc* be a coverage graph, *Gc** be the graph transformed from *Gc* by the node-disjoint transformation, and *Fc* be the minimum cost maximum flow plan on *Gc**. The node set *Vd* associated with *Fc* is the minimum set having the maximum number of node-disjoint traversable paths on *Gc*.

Proof: We first show the traversable paths of *Vd* are node-disjoint as follows. It is impossible to have more than one flow going through a node on *Gc* for the following two reasons: (1) each edge on *Gc* is with Capacity=0 or Capacity=1, and (2) each node on *Gc* with multiple inbound edges and multiple outbound edges is transformed into two virtual nodes in *Gc** with one in-between edge of Capacity=1. The traversable paths of *Vd* are thus node-disjoint.

We then show that the number of traversable paths of *Vd* is maximized. This is trivial since *Vd* is associated with *Fc*, and *Fc* has the maximum number of flows, each of which corresponds to one traversal path on *Gc*.

Finally, we show *Vd* is minimized. As we have just shown, at most one flow goes through a node on *Gc*. Moreover, ONSA assigns Cost 0 to the edges between virtual nodes in the node-edge transformation and to the edges going to the target node *T*, it assigns Cost 1 to other edges. Hence, the total cost of *Fc* is the cardinality of *Vd*. Since the total cost is minimized, *Vd* is minimized. □

**Theorem 2**. Let *Gt* be a transmission graph, *Gt** be the graph transformed from *Gt* by the node-edge transformation, and *Ft* be the minimum cost maximum flow plan on *Gt** with a given set *Vd* of detecting nodes and a given set *Vk* of sink nodes. The node set *Vf* associated with *Ft* is the minimum set to make $Vd \cup Vf$ sink-connected on *Gt*.

Proof: We first show $Vd \cup Vf$ is sink connected. In *Ft*, we can find a flow going through a detecting node in *Vd*, and 0 or more nodes in *Vf*, towards one of the sink nodes for the following reasons: (1) an edge with Capacity=1 is added to connect the virtual source node *S* and every node in *Vd*, (2) an edge with Capacity=$\infty$ is added to connect a sink node in *Vk* and the virtual target node *T*, and (3) all other edges are with Capacity=$\infty$. We thus have $Vd \cup Vf$ is sink connected.

We then show *Vf* is minimized. Since each node in *Gt* is transformed into two virtual nodes with an in-between edge of Capacity=$\infty$ and Cost=1, and all other edges are with Cost=0, we have the total cost of Ft is the cardinality of $Vd \cup Vf$. Since the total cost of *Ft* is minimized, $Vd \cup Vf$ is minimized and *Vf* is minimized as well. □

### 4.3 Time Complexity Analysis

The time complexity of ONSA is dominated by Step 3 and Step 7, which execute the maximum flow minimum cost algorithm on *Gc** and *Gt**, respectively. The maximum flow minimum cost algorithm is actually the combination of the Edmonds-Karp algorithm [Cormen et al. 2001], which is of $O(|V| \cdot |E|^2)$ time complexity for a graph of vertex set *V* and edge set *E*, and the Orlin-Ahuja algorithm [Orlin and Ahuja 1992], which is of $O( |E| \cdot \log |V| (|E| + |V| \cdot \log |V|) )$ time complexity. The time complexity of ONSA is thus $O( |Ec^*| \cdot \log |Vc^*| (|Ec^*| + |Vc^*| \cdot \log |Vc^*|) + |Et^*| \cdot \log |Vt^*| (|Et^*| + |Vt^*| \cdot \log |Vt^*|))$, where *Vc** (resp., *Vt**) is the cardinality of the vertex set in *Gc** (resp., *Gt**), and *Ec** (resp., *Et**) is the cardinality of the edge set in *Gc** (resp., *Gt**). To execute the optimization computation of ONSA will consume some computation power and memory storage. Fortunately, as we have mentioned earlier, ONSA is performed by the backend system, which is assumed to have unlimited power supply and enormous computing power. All the sensor nodes in the WSN only need to collaborate to deliver/forward their local information required by ONSA to the sink nodes, which in turn forward the information to the backend system. In other words, ONSA does not impose much computation and memory consumption on normal sensor nodes.

## 5. SIMULATION

We conduct simulation experiments for the proposed ONSA algorithm, and compare the simulation results with those of the *global determination algorithm (GDA)*, which is proposed in [ Kumar, 2003], for determining the highest degree of barrier coverage by using the maximum flow algorithm [Cormen et al. 2001]. Since GDA does not consider the sink-connected property, we use the maximum flow algorithm for GDA to select extra sensor nodes to serve as forwarding nodes to endow GDA with the property. In this way, GDA and ONSA can both achieve the sink-connected barrier coverage with the highest degree. To evaluate the performances of ONSA and GDA in WSNs, we used the NS2 simulation tool [NS2, 2012] along with the 802.15.4 module developed by Zheng and Lee [Zheng and Lee, 2004]. Moreover, the Matlab [Matlab, 2012] software is used to execute the maximum flow minimum cost algorithm.

Table 1. Simulation Settings

| | |
|---|---|
| Network Dimension | 120m x 10m |
| Network Interface | 802.15.4 unslotted CSMA/CA |
| Network Bandwidth | 250 kbps |
| Sensing Range | 10m |
| Transmission Range | 10m |
| Simulation Duration | 10s |
| No. of Deployed Nodes | 150, 200, 250, or 300 |

| | |
|---|---|
| Traffic Type | CBR (constant bit rate) |
| Sending Frequency | 1 packet/sec |
| Packet Size | 70 bytes |
| Transmitting Power | 19.8 mW |
| Receiving Power | 35.5 mW |
| Idling Power | 0.8 mW |
| No. of Experiments | 100 times/case |

The parameter settings used in simulation are listed in Table 1 and described as follows. All the sensors are equipped with 802.15.4 unslotted CSMA/CA network interface, and are randomly deployed in a 120m x 10m rectangle-shaped area. The number of nodes is 150, 200, 250, or 300. The sensing area coverage radius (sensing range) and the wireless transmission radius (transmission range) are both set to 10m. The power of the radio module in different mode is set according to the commercial transceiver CC2420 [TI, 2012]. The duration of a simulation experiment is 10 seconds. The states of nodes and links are assumed to be fixed during the simulation duration. The result of a simulation case is derived by averaging 100 experiment results.

We first consider the case that only 1 sink node is located at (60m, 5m), the center position of the monitored rectangle. We compare ONSA and GDA in terms of the number of nodes selected to achieve the sink-connected barrier coverage with the highest degree. As shown in Fig. 9, ONSA selects fewer nodes than GDA for all the numbers of deployed nodes.

We then consider the case that 2 sink nodes are respectively located at (40m, 5m) and (80m, 5m) which is relative to the leftmost and lowest position of the rectangle. By Fig. 10 we can observe that more sink nodes lead to fewer selected nodes and that ONSA again selects fewer nodes than GDA. We conclude that ONSA needs fewer nodes than GDA to achieve sink-connected barrier coverage with the highest degree. This is because ONSA, which is based on the maximum flow minimum cost algorithm, will always select the minimum number of nodes.

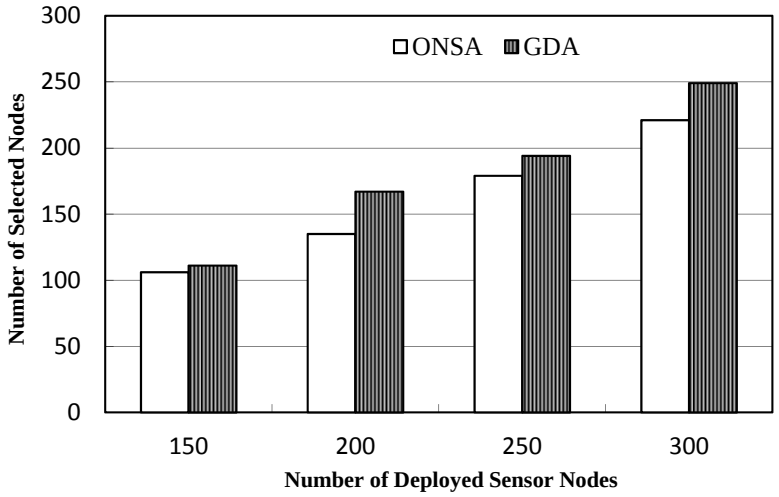



Fig. 9. Comparisons of ONSA and GDA with 1 sink node in terms of the number of selected nodes

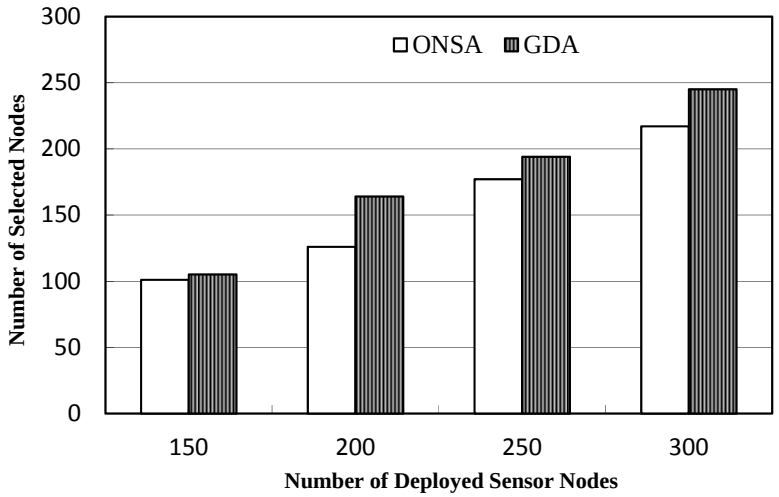



Fig 10. Comparisons of ONSA and GDA with 2 sink nodes in terms of the number of selected nodes

We compare ONSA and GDA in terms of energy consumptions in transmissions. The simulations are conducted by simulating two event transmission scenarios: (i) one detection event source, and (ii) two detection event sources. The first (resp., second) scenario is simulated by randomly selecting one (resp., two) source node(s) to transmit 10 notification packets of 70 bytes to one of the sink nodes at the rate of one packet per second. The packet transmission is based on the path determined by the maximum flow minimum cost algorithm for the cases of 1 sink node and 2 sink nodes. As shown in Figs. 11 and 12, the energy consumption of ONSA is lower than GDA in both scenarios for both cases of 1 sink node and 2 sink nodes. This is because more nodes are selected by GDA than ONSA to turn on their sensing module and/or radio module. We can also observe that larger numbers of deployed sensor nodes leads to more energy consumption. This is because more nodes are selected to be active to construct barrier coverage with higher degrees. The cases of two sink nodes have less energy consumption than the cases of one sink node. This is because the former cases have smaller source-to-sink hop counts than the latter.

We also compare ONSA and GDA in terms of the packet delay of transmitting an event notification packet from a source node to a sink node. The comparisons results of packet delay are reported in Figs. 13 and 14, which show that the packet delay is almost the same between ONSA and GDA for both scenarios of one detection source or two detection sources and that the delay does not vary with the number of deployed nodes. This is because both ONSA and GDA use pre-established paths with fixed hop counts to transmit the packet. We can also observe that the cases of 2 sink nodes have smaller delay than the cases of 1 sink node. This is because the former cases have smaller source-to-sink hop counts than the latter.

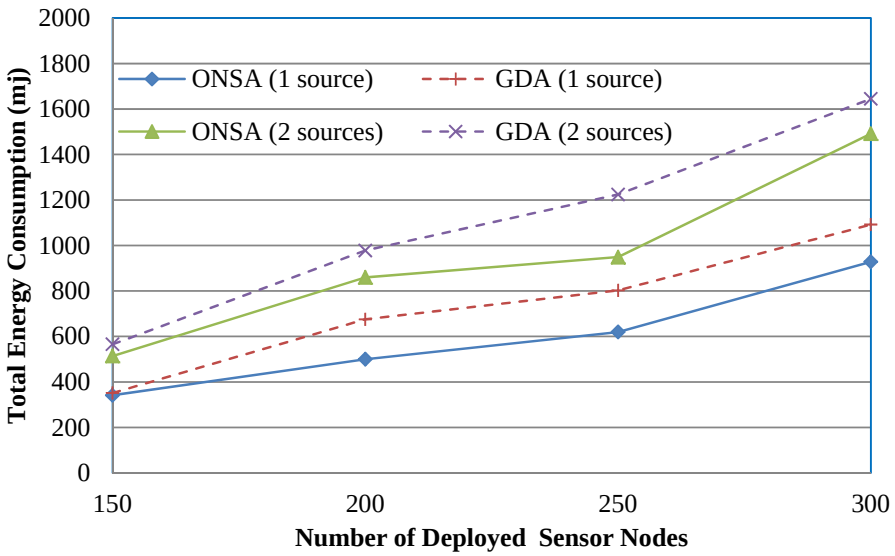



Fig. 11. Comparisons of ONSA and GDA with 1 sink node in terms of the total energy consumption

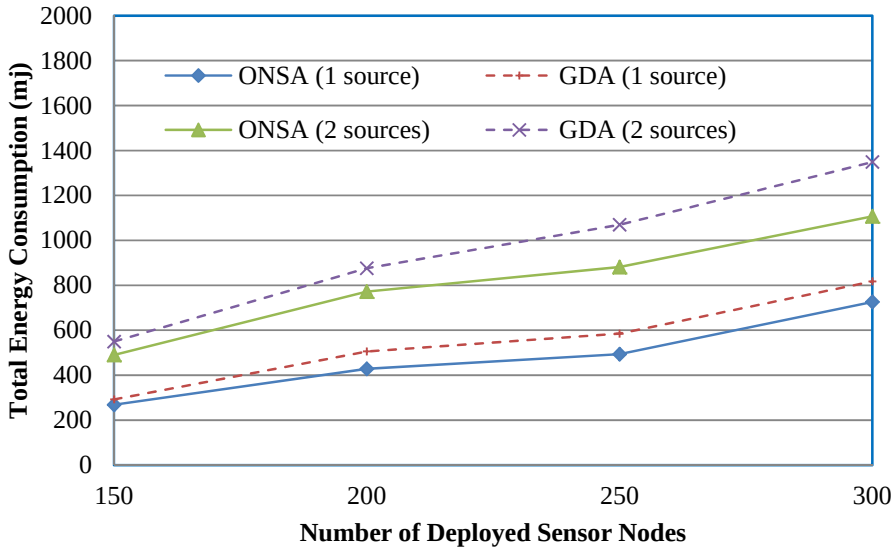



Fig. 12. Comparisons of ONSA and GDA with 2 sink nodes in terms of the total energy consumption

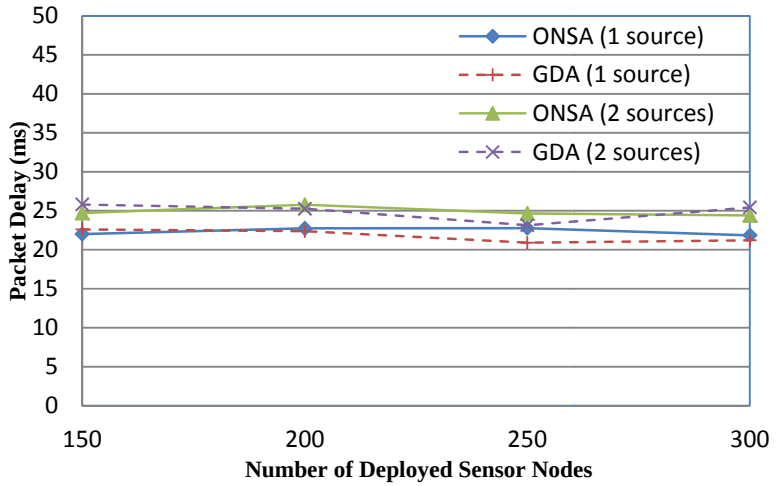



Fig. 13. Comparisons of ONSA and GDA with 1 sink node in terms of the packet delay

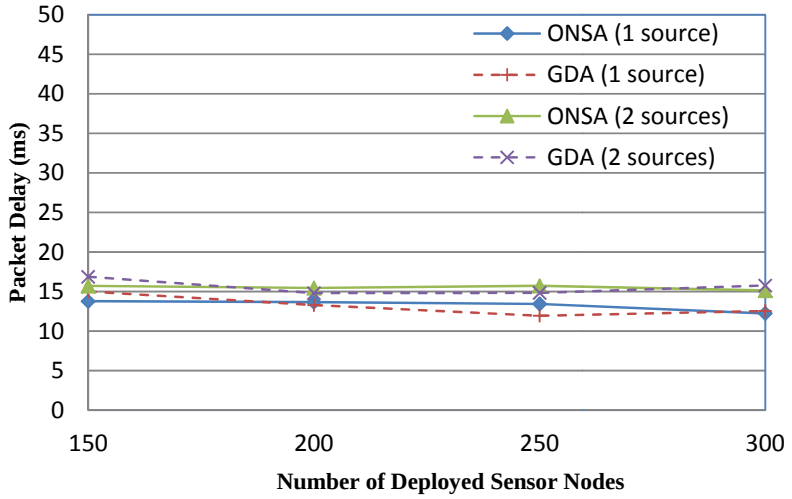



Fig. 14. Comparisons of ONSA and GDA with 2 sink nodes in terms of the packet delay

## 6. CONCLUSION

In this paper, we studied the sink-connected barrier coverage problem to achieve two goals: (1) to maximize the degree of barrier coverage by using the minimum number of detecting nodes and (2) to minimize the number of forwarding nodes to hold the sink-connected property. A maximum-flow-minimum-cost based algorithm, called the optimal node selection algorithm (ONSA), is proposed to solve the problem. ONSA is based two transformations (i.e., the node-disjoint and the node-edge transformations) and Edmonds-Karp algorithm and Orlin-Ahuja algorithm. The time complexity of ONSA is polynomial and is equal to that of the Orlin-Ahuja algorithm. We also perform simulation experiments for ONSA and a related algorithm called global determination algorithm (GDA). The simulation results show that ONSA and GDA have similar packet transmission delay and that ONSA is better than GDA in terms of the number of selected nodes and the total energy consumption.

Given the node position information and the topology of a WSN, ONSA can be used to construct optimal sink-connected barrier-coverage. When the WSN topology changes due to node/link failure or insertion, the topology changes should be detected to initiate some actions for recover the barrier-coverage. All active nodes are demanded to report its state and associated link states to the sink node periodically. On detecting node and/or link condition changes, the sink node then broadcasts requests to all nodes, either active or inactive, to collect the topology information and make the ONSA run at the backend system to determine new barrier coverage. Note that we assume inactive nodes turn on their radio modules periodically to guarantee to receive broadcast requests within a pre-specified period of time. Afterwards, the new coverage information is disseminated to all nodes to form the new barrier coverage.

**REFERENCES**


R. K. Ahuja, T. L. Magnanti, and J. B. Orlin, *Network Flows: Theory, Algorithms, and Applications*, Prentice Hall, Upper Saddle River, New Jersey, 1993.

P. Balister, B. Bollobas, A. Sarkar, and S. Kumar, "Reliable density estimates for coverage and connectivity in thin strips of finite length," in *Proceedings of ACM Mobicom*, pp. 75-86, 2007.

M. Cardei and J. Wu, "Energy-efficient coverage problems in wireless ad hoc sensor networks," *Computer Commun.*, vol. 29, no. 4, pp. 413–420, 2006.

A. Chen, S. Kumar, and T.-H. Lai, "Designing localized algorithms for barrier coverage," in *Proceedings of ACM Mobicom*, pp. 63-74, 2007.

A. Chen, T.-H. Lai, and D. Xuan, "Measuring and guaranteeing quality of barrier-coverage in wireless sensor networks," in *Proceedings of The ACM International Symposium on Mobile Ad Hoc Networking and Computing (MobiHoc)*, pp. 421-430, 2008.

T.-H. Cormen, C. E. Leiserson, R. L. Rivest, and C. Stein, *Introduction to Algorithms (Second Edition)*, MIT Press and McGraw-Hill, 2001.

V. Coskun, "Relocating sensor nodes to maximize cumulative connected coverage in wireless sensor networks," *Sensors*, vol. 8, no. 4, pp. 2792-2817, 2008.

D. Gage, "Command control for many-robot systems," in *Proceedings of the Nineteenth Annual AUVS Technical Symposium (AUVS-92)*, pp. 28-34, 1992.

M. R. Garey, and D. S. Johnson, *Computer and Intractability*, W. H. Freeman and Co., 1979.

C.-F. Huang, and Y.-C. Tseng, "A survey of solutions to the coverage problems in wireless sensor networks," *Journal of Internet Technology*, vol. 6, no. 1, pp. 1–8, 2005.

J.-R. Jiang, and T.-M. Sung, "Energy-Efficient Coverage and Connectivity Maintenance for Wireless Sensor Networks," *Journal of Networks*, vol. 4, no. 6, pp. 403-410, 2009.

S. Kumar, T.-H. Lai, and A. Arora, "Barrier coverage with wireless sensors," *Wireless Networks*, vol. 13, no. 6, pp. 817–834, 2007.

J. Li, L. Andrew, C. H. Foh, M. Zukerman, and H.-H. Chen, "Connectivity, coverage and placement in wireless sensor networks," *Sensors*, vol. 9, no. 10, pp. 7664-7693, 2009.

B. Liu, and D. Towsley, "A study on the coverage of large-scale sensor networks," in *Proceedings of the 1st IEEE International Conference on Mobile Ad-hoc and Sensor Systems*, pp. 475-483, 2004.

J. B. Orlin, and R.K. Ahuja, "New scaling algorithms for assignment and minimum cycle mean problems," *Mathematical Programming*, vol. 54, pp. 41-56, 1992.

A. Saipulla, C. Westphal, B. Liu, and J. Wang, "Barrier coverage of line-based deployed wireless sensor networks," in *Proceedings of IEEE INFOCOM 2009*, pp. 127-135, Brazil, 2009.

B. Wang, "Coverage problems in sensor networks: A survey," *ACM Computing Surveys*, vol. 43, issue 4, pp. 32:1-32:53, October 2011.

Y. Wang, and G. Cao, "Barrier coverage in camera sensor networks," in *Proceedings of ACM MobiHoc'11*, 2011.

H. Zhang, and J. Hou, "Maintaining sensing coverage and connectivity in large sensor networks," *Ad Hoc and Sensor Wireless Networks*, vol. 1, no. 1-2, pp. 89-124, 2005.

Q. Zhao, and M. Gurusamy, "Lifetime Maximization for Connected Target Coverage in Wireless Sensor Networks," *IEEE/ACM Transactions on Networking*, vol.16, no.6, pp.1378-1391, Dec. 2008.

J. Zheng, and M. J. Lee, *A comprehensive performance study of IEEE 802.15.4*, IEEE Press Book, 2004.

Network Simulator NS2. url: http://www.isu.edu/nsnam/ns. (Accessed on March 3, 2013).

Matlab, url: http://www.mathworks.com. (Accessed on March 3, 2013).

Data Sheet for CC2420 2.4GHz IEEE 802.15.4 RF transceiver, url:

http://www.ti.com/lit/ds/symlink/cc2420.pdf. (Accessed on March 3, 2013)