

Expected Quorum Overlap Sizes of Quorum Systems for Asynchronous Power-Saving in Mobile Ad Hoc Networks

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Abstract

Quorum systems satisfying the rotation closure property can be used to realize asynchronous power-saving algorithms for mobile ad hoc networks. The FPP, grid, cyclic, torus and e-torus quorum systems can provide the algorithms with the lowest or near lowest active ratios since they have the optimal or near optimal quorum sizes. The algorithms guarantee that a node can sense the status of every neighbor by receiving one or more beacons from it within a round of beacon intervals. Traditionally, the smallest quorum overlap size (SQOS) and the maximum quorum overlap separation (MQOS) are used to measure the neighbor sensibility. However, it is difficult to differentiate the quorum systems by SQOS and MQOS since most of them have the same SQOS and MQOS values. In this paper, the expected quorum overlap size (EQOS) is proposed as an average-case neighbor sensibility measurement. We can easily judge the goodness of quorum systems by EQOS since they have different EQOS values. Larger-than-one EQOS values are desirable. Observing quorum systems are of EQOS values far larger than one, we are inspired to devise a new quorum system, called the fraction torus (f-torus) quorum system, for the construction of flexible mobility-adaptive power-saving algorithms. The f-torus quorum system can further reduce the active ratio to save energy by shrinking the quorum size, while still keeping the EQOS larger than one. We derive EQOS values for all the above-mentioned quorum systems by analysis and simulation experiments. As we will show, the EQOS analysis and simulation results coincide very closely.

Key words: mobile ad hoc network (MANET), asynchronous power-saving algorithm, optimal quorum system

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1 Introduction

A *quorum system* is a collection of sets, called *quorums*, with the *intersection property* that any two quorums overlap mutually. Quorum-based algorithms are useful for solving many problems, such as mutual exclusion [1, 2] and replica control [3, 4] in conventional distributed systems, as well as location management [5, 6], information dissemination/retrieval [7, 8] and data aggregation [9] in mobile ad hoc networks (MANETs) and/or wireless sensor networks (WSNs). The basic idea of the quorum-based algorithms is as follows. System nodes are first grouped as quorums to form a quorum systems. A node then accesses and/or locks all nodes of a quorum to perform the desired operation. As shown in [1, 3, 5, 6, 8], quorum-based algorithms usually have low control overheads and high degrees of fault-tolerance when compared to related ones.

Unlike other quorum-based algorithms, the algorithms in [10] take time intervals, instead of system nodes, as elements of a quorum to achieve asynchronous power-saving management in IEEE 802.11-based MANETs. A MANET consists of a set of mobile nodes, and has no support of any infrastructure. Nodes can communicate with each other by multi-hop radio transmission. When there is no datum to send/receive, a node may tune its radio into *power saving (or doze)* mode to save energy since nodes are usually sustained by limited-capacity batteries and the radio component consumes major energy. However, a power-saving node should turn on its radio periodically to check if there are data to receive. To achieve this, the time axis is divided into equal-length *beacon intervals*. Consecutive n intervals are regarded as a *round*, with each being labeled as $0, \dots, n - 1$. A quorum system is generated from the universal set $\{0, \dots, n - 1\}$. A node chooses one of the quorums and regard a beacon interval as a *quorum interval* if its associated label is a member of the chosen quorum. In a beacon interval, a node keeps its radio active and sends a *beacon* signal to notify neighboring nodes of its existence. On the contrary, it tunes the radio into power saving mode to save energy in non-quorum intervals. Since beacons are usually embedded with useful information, such as nodes' locations or statuses, it is desirable for a node to receive beacons from every neighboring node. As shown in [10], the quorum systems satisfying the *rotation closure* property can be applied to realize asynchronous power-saving algorithms to ensure that every pair of neighboring nodes can receive each other's beacon once per round, and can then exchange data properly even when their clocks are asynchronous.

Quorum system characteristics can be used to measure the performance of traditional quorum-based algorithms. For example, the *quorum size* is used to evaluate the control overhead of an algorithm; the *availability*, the overall probability that all members of a quorum are available, is used to measure the degree of fault tolerance [11]; the *load*, the probability that the busiest node

is accessed, is used to estimate the level of load balance [12]. Quorum system characteristics can be used to choose proper quorum systems for specific applications. For example, the paper [1] suggests using cohorts quorum systems to realize mutual exclusion algorithms, since cohorts quorum systems have a constant quorum size to achieve the constant overhead in the best case, and have a good availability to achieve comparably high degree of fault-tolerance.

The performance of quorum-based power-saving algorithms can also be measured by quorum system characteristics. For example, the *active ratio*, which is the ratio that a node has to keep its radio active, can be measured by the ratio of quorum size to the round size. As shown in [10], the finite projective plane (FPP) [13], grid [13], cyclic [14], torus [15] and extended torus (e-torus) [10] quorum systems satisfy the rotation closure property and have the optimal or near optimal active ratios. They are thus called the optimal or near optimal quorum systems in this paper. For another example, *neighbor sensibility*, can be measured by the *maximum quorum overlap separation (MQOS)* for evaluating the maximum delay for a node to be aware of a newly arriving neighboring node [10]. Neighbor sensibility can also be measured by the *smallest quorum overlap size (SQOS)* for evaluating the minimal number of beacons heard in a round of beacon intervals. However, it is hard to judge the goodness of quorum systems by MQOS or SQOS since most quorum systems have the same MQOS and SQOS values [10].

In this paper, we propose using the *expected quorum overlap size (EQOS)* of quorum systems to help evaluate the average case neighbor sensibility. We analyze EQOS for the optimal or near optimal quorum systems. With the help of the analyzed results, we can easily select proper quorum systems to construct power saving algorithms for specific environments. Traditional quorum systems have the property that the SQOS is at least one to guarantee the correctness of algorithms. However, we observe that quorum-based power-saving algorithms still work well even without the property. The observation inspires us to design a new quorum system, called the *fraction torus (f-torus)* quorum system. The f-torus quorum system is very flexible and can be used for the construction of mobility-adaptive asynchronous power saving algorithms. We analyze the f-torus quorum system to show that it has comparably high EQOS values and can achieve even lower active ratios than the FPP, grid, cyclic, torus and e-torus quorum systems. We also perform simulation experiments to derive EQOS values for all the above-mentioned quorum systems. As we will show, the analysis and simulation results coincide very closely.

The rest of this paper is organized as follows. Some preliminaries are given in Section 2. Section 3 defines EQOS formally and derive EQOS formulas for the optimal and near optimal quorum systems. In Section 4, we analyze and compare the optimal and near optimal quorum systems in terms of EQOS, MQOS, SQOS, and so on. Analyzed EQOS values are also compared with

simulated values in this section. We propose the f-torus quorum system, and analyze and simulate its EQOS in Section 5. At last, some concluding remarks are drawn in Section 6.

2 Preliminaries

2.1 Quorum-Based Power-Saving Algorithms for MANETs

IEEE 802.11 supports two power modes: *active* and *power-saving (PS)*. Under the PS mode, a node can reduce its radio activity by only monitoring periodical beacon signals. Tuning a node to the PS mode can save a lot of energy. For example, ORiNOCO IEEE 802.11b PC Gold Card [16] consumes 1400mW, 950mW and 805mW when it is transmitting, receiving, and monitoring data packets, respectively. However, under the PS or the doze mode, it consumes only 60mW.

In [10, 17], the time axis is divided into equal-length beacon intervals. Each node label the intervals as $0, \dots, n - 1$ cyclical and groups them into rounds such that each round consists of n consecutive intervals. The node generates a grid quorum system [13] under the universal set $\{0, \dots, n - 1\}$. It then pick a quorum from the grid quorum system and picks quorum intervals accordingly. For example, if each round consists of 4 intervals, then we have an associated grid quorum system: $\mathcal{Q} = \{\{0, 1, 2\}, \{0, 1, 3\}, \{0, 2, 3\}, \{1, 2, 3\}\}$ under the universal set $U = \{0, 1, 2, 3\}$. If the quorum $\{0, 1, 2\}$ is picked, then the quorum intervals will be those labeled by 0, 1, or 2. In a quorum interval, a node will issue a beacon signal and keeps in the active mode; in a non-quorum interval, a node can turn to the power saving mode to save energy. As shown in [10], any quorum system satisfying the rotation closure property can be translated to an asynchronous power-saving algorithm for mobile ad hoc networks to ensure that in a round of n consecutive intervals, each node can receive at least one beacon signal from any of its neighboring nodes. Since beacon signal carries information necessary for wireless communication, a node is thus aware of the existence of any neighboring node and can communicate with it properly.

Below, we introduce the concepts of the quorum system and the rotation closure property.

Definition 1 *Given a universal set $U = \{0, \dots, n - 1\}$, a quorum system \mathcal{Q} under U is a collection of non-empty subsets of U , each called a quorum, which satisfies the intersection property:*

$$\forall G, H \in \mathcal{Q} : G \cap H \neq \emptyset.$$

For example, $\mathcal{Q} = \{\{0, 1\}, \{0, 2\}, \{1, 2\}\}$ is a quorum system under $U = \{0, 1, 2\}$.

Definition 2 Given a non-negative integer i and a quorum H in a quorum system \mathcal{Q} under $U = \{0, \dots, n-1\}$, we define $rotate(H, i) = \{(j+i) \bmod n \mid j \in H\}$.

Definition 3 A quorum system \mathcal{Q} under $U = \{0, \dots, n-1\}$ is said to have the rotation closure property if

$$\forall G, H \in \mathcal{Q}, i \in \{0, \dots, n-1\} : G \cap rotate(H, i) \neq \emptyset.$$

For instance, the quorum system $\mathcal{Q} = \{\{0, 1\}, \{0, 2\}, \{1, 2\}\}$ under $\{0, 1, 2\}$ has the rotation closure property. However, the quorum system $\mathcal{Q}' = \{\{0, 1\}, \{0, 2\}, \{0, 3\}, \{1, 2, 3\}\}$ under $\{0, 1, 2, 3\}$ has no rotation closure property because $\{0, 1\} \cap rotate(\{0, 3\}, 3) = \emptyset$.

The paper [10] also derives a lower bound of the quorum size for any quorum system satisfying the rotation closure property. As shown in [10], a quorum system \mathcal{Q} under $\{0, \dots, n-1\}$ satisfying the rotation closure property has the quorum size lower bound s , where $s(s-1) + 1 = n$ and $s-1$ is a prime power (note that $s \approx \sqrt{n}$). Since smaller quorums imply lower active ratios (less energy expenditures), we should always concentrate on quorum systems owning the optimal or near optimal quorum sizes. The paper [10] has identified a group of quorum systems that satisfy the rotation closure property and have optimal (the finite projective plane (FPP) quorum system [13]) or near optimal (the grid quorum system [13], the torus quorum system [15], and the cyclic quorum system [14]) quorum sizes. It also proposes a novel e-torus(k) quorum system to be translated to an adaptive power saving algorithm. The algorithm ranks a node's mobility into k levels. Whenever a node determines that its mobility falls within level i ($1 \leq i \leq k$), it adjusts its quorum intervals based on any e-torus(i) quorum so that a node can dynamically adjust its sensibility to the environment change in its neighborhood. The e-torus(k) quorum system is actually a torus quorum system when $k = 1$ and thus has a near optimal quorum size; it's quorum size grows gradually as k increases.

Neighbor sensibility is an important measurement of asynchronous quorum-based power-saving algorithms. It can be measured by the smallest quorum overlap size (SQOS) to evaluate the minimum number of beacons received in a round of beacon intervals. However, it is hard to differentiate quorum systems by SQOS since most quorum systems have identical SQOS values. For example, the grid quorum system has SQOS of value 2, while other optimal or near optimal quorum systems have SQOS of value 1. Neighbor sensibility can also be measured by the maximum quorum overlap separation (MQOS) for estimating the longest delay for a PS node to detect the beacon of a newly

approaching PS node in its neighborhood. Below, we give the formal definition of MQOS.

Definition 4 For a quorum system \mathcal{Q} under $U = \{0, \dots, n-1\}$, the maximum quorum overlap separation (MQOS) of \mathcal{Q} is $\text{Max}(d(i, j))$, where $i, j \in (G \cap H)$, $G, H \in \mathcal{Q}$, and $d(i, j) = \text{Min}((i - j) \bmod n, (j - i) \bmod n)$.

The optimal or near optimal quorum systems have MQOS values of n except that the grid quorum system has the MQOS value of $(n - \sqrt{n} + 1)$ [10]. Therefore, it is also difficult to differentiate quorum systems by MQOS.

3 The EQOS of Quorum Systems

3.1 The Definition of EQOS

In this section, we propose using the expected quorum overlap size (EQOS) of quorum systems for evaluating the neighbor sensibility of power-saving algorithms. The formal definition of EQOS is given in the following.

Definition 5 For a quorum system \mathcal{Q} under $U = \{0, \dots, n-1\}$, the expected quorum overlap size (EQOS) of \mathcal{Q} is

$$\sum_{G, H \in \mathcal{Q}} p(G)p(H)|G \cap H|,$$

where $P(G)$ and $P(H)$ is respectively the probability of accessing quorums G and H for a quorum access strategy.

EQOS can be used as an average-case estimation of neighbor sensibility and can help us differentiate quorum systems since they have different EQOS values. Below, we analyze the EQOS for optimal or near optimal quorum systems satisfying the rotation closure property.

3.2 The EQOS of Grid Quorum Systems

The grid quorum system [13] arranges elements of the universal set $U = \{0, \dots, n-1\}$ as a $\sqrt{n} \times \sqrt{n}$ array. A quorum can be any set containing a full column plus a full row of elements in the array. The grid quorum system has a near optimal quorum size of $2\sqrt{n} - 1$.

Suppose Q and Q' are two quorums in the grid quorum system, where Q contains full column c plus full row r of elements and Q' contains full column

c' plus full row r' of elements. Note that it is possible that $Q = Q'$. Therefore, there are n^2 possible permutations of Q and Q' . The EQOS of the grid quorum system can be figured out by considering the following four independent cases of Q and Q' :

Case 1. $r \neq r'$ and $c \neq c'$: The overlap of Q and Q' has size 2 and there are $n \cdot (\sqrt{n} - 1) \cdot (\sqrt{n} - 1)$ occurrences for such a case.

Case 2. $r = r'$ and $c \neq c'$: The overlap of Q and Q' has size \sqrt{n} and there are $n \cdot (\sqrt{n} - 1)$ occurrences for such a case.

Case 3. $r \neq r'$ and $c = c'$: The overlap of Q and Q' has size \sqrt{n} and there are $n \cdot (\sqrt{n} - 1)$ occurrences for such a case.

Case 4. $r = r'$ and $c = c'$: The overlap of Q and Q' has size $2\sqrt{n} - 1$ and there are n occurrences for such a case.

Summing up the products of overlap sizes and occurrence probabilities for all cases, we have the EQOS of Q and Q' . Thus, the EQOS of the grid quorum system under the universal set $\{0, \dots, n - 1\}$ is

$$\begin{aligned} & \frac{2n \cdot (\sqrt{n} - 1) \cdot (\sqrt{n} - 1) + 2\sqrt{n} \cdot n \cdot (\sqrt{n} - 1) + (2\sqrt{n} - 1) \cdot n}{n^2} \\ &= \frac{4n - 4\sqrt{n} + 1}{n} = \frac{(2\sqrt{n} - 1)^2}{n} \end{aligned}$$

3.3 The EQOS of Torus Quorum Systems

Similar to the grid quorum system, the torus quorum system [15] also adopts an array structure. The universal set is arranged as a $t \times w$ array, where $tw = n$. Following the concept of torus, the rightmost column (resp., the bottom row) in the array are regarded as wrapping around back to the leftmost column (resp., the top row). A quorum is formed by picking any column c , $0 \leq c \leq w - 1$, plus $\lfloor w/2 \rfloor$ elements, each of which falls in any position of column $c + i, i = 1.. \lfloor w/2 \rfloor$. Fig. 1 illustrates the construction of two torus quorums G and H under $U = \{0, \dots, 17\}$ with $t = 3$ and $w = 6$. G is formed by picking the second column plus three elements, each from one of the third, fourth, and fifth columns. H is formed by picking the sixth column plus three elements, each from one of the first, second, and third columns. G and H intersect at element 7.

As shown in [15], if we let $t = w/2$, the quorum size will be approximately $\sqrt{2tw} = \sqrt{2n}$, which is near optimal. Below, we analyze the EQOS for such torus quorum systems.

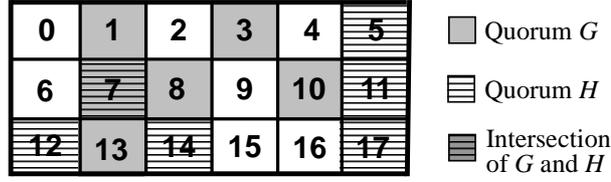


Fig. 1. Two quorums of the torus quorum system in a 3×6 torus.

Suppose Q and Q' are two quorums in the torus quorum system, where Q contains all elements of column c plus one element from each of columns $c + i, i = 1.. \lfloor w/2 \rfloor$ and Q' contains all elements of column c' plus one element from each of columns $c' + i, i = 1.. \lfloor w/2 \rfloor$. It is noted that we define d , the distance of c and c' , to be $\text{Min}((c - c') \bmod w, (c' - c) \bmod w)$. The EQOS of the torus quorum system can be figured out by considering the following three independent cases of Q and Q' :

Case 1. $c = c'$: The overlap of Q and Q' is expected to have size $t + (1/t) \cdot \lfloor w/2 \rfloor$ and there are w occurrences for such a case.

Case 2. $c \neq c'$ and $1 \leq d < \lfloor w/2 \rfloor$: The overlap of Q and Q' is expected to have size $1 + (1/t) \cdot (\lfloor w/2 \rfloor - d)$ and there are $2w$ occurrences for such a case for each $d, 1 \leq d < \lfloor w/2 \rfloor$.

Case 3. $c \neq c'$ and $d = \lfloor w/2 \rfloor$: The overlap of Q and Q' has size 2 and there are w occurrences for such a case if w is even. Otherwise (w is odd), the overlap of Q and Q' has size 1 and there are $2w$ occurrences for such a case.

There are w^2 possible permutations of Q and Q' . Thus, the EQOS of the torus quorum system under $\{0, \dots, n-1\}$ is

$$\frac{w \cdot (t + \frac{\lfloor w/2 \rfloor}{t}) + 2w \cdot \sum_{d=1}^{\lfloor w/2 \rfloor - 1} [1 + \frac{\lfloor w/2 \rfloor - d}{t}] + 2w}{w^2}$$

$$= \frac{(t + \frac{\lfloor w/2 \rfloor}{t}) + 2(\lfloor w/2 \rfloor - 1)(1 + \frac{\lfloor w/2 \rfloor}{2t}) + 2}{w}$$

By substituting $w = 2t$, we have EQOS of the torus quorum system is

$$\frac{(t + 1) + 2(t - 1)(1 + \frac{1}{2}) + 2}{2t} = 2$$

3.4 The EQOS of Cyclic Quorum Systems

The cyclic quorum systems [14] are constructed from the *difference sets* as defined below.

Definition 6 A subset $D = \{d_1, d_2, \dots, d_k\}$ of Z_n is called a *difference set* under Z_n if for every $e \neq 0 \pmod{n}$ there exist elements d_i and $d_j \in D$ such that $d_i - d_j = e \pmod{n}$.

Definition 7 Given any difference set $D = \{d_1, d_2, \dots, d_s\}$ under Z_n , the cyclic quorum system defined by D is $\mathcal{Q} = \{G_1, G_2, \dots, G_n\}$, where $G_i = \{d_1 + i, d_2 + i, \dots, d_s + i\} \pmod{n}$, $i = 0, \dots, n - 1$.

For example, $D = \{0, 1, 2, 4\} \subseteq Z_8$ is a difference set under Z_8 since each $e = 1..7$ can be generated by taking the difference of two elements in D . Given D , $\mathcal{Q} = \{G_0 = \{0, 1, 2, 4\}, G_1 = \{1, 2, 3, 5\}, G_2 = \{2, 3, 4, 6\}, G_3 = \{3, 4, 5, 7\}, G_4 = \{4, 5, 6, 0\}, G_5 = \{5, 6, 7, 1\}, G_6 = \{6, 7, 0, 2\}, G_7 = \{7, 0, 1, 3\}\}$ is a cyclic quorum system under Z_8 .

Given any n , a difference set as small as s can be found when $s(s-1)+1 = n$ and $s-1$ is a prime power. Such a difference set is called the *Singer difference set* [18]. For example, the sets $\{1, 2, 4\}$ under Z_7 and $\{1, 2, 4, 9, 13, 19\}$ under Z_{31} are Singer difference sets. As shown in [10], in this case the quorum size s meets the lower bound. So cyclic quorum systems defined by the Singer difference sets are optimal in terms of the quorum size. Reference [14] had conducted exhausted searches to find the minimal difference sets under Z_n for $n = 4..111$. The results are useful here to construct near-optimal cyclic quorum systems.

Below, we analyze the EQOS of the cyclic quorum system based on a stricter difference set, the λ -difference set, as defined below.

Definition 8 A subset $D = \{d_1, \dots, d_s\}$ of Z_n is called a λ -*difference set* under Z_n if for every $e \neq 0 \pmod{n}$ there exist exactly λ ordered pairs (d_i, d_j) , where $d_i, d_j \in D$, such that $d_i - d_j = e \pmod{n}$.

For example, the set $D = \{0, 1, 2, 4, 5, 8, 10\} \subseteq Z_{15}$ is a 3-difference set under Z_{15} since for each integer $e \in \{1, \dots, 14\}$, there exist exactly three ordered pairs of elements of D to generate e (for example, the pairs $(1,0)$, $(2,1)$ and $(5,4)$ generate 1).

The cyclic quorum system \mathcal{Q} constructed on the basis of λ -difference set has the property that every pair of different quorums has the overlap of size λ . Suppose Q and Q' are two randomly selected quorums in \mathcal{Q} . It is easy to derive that there are totally $\binom{n+1}{2}$ possible combinations (not permutations)

of Q and Q' . The EQOS of \mathcal{Q} can be analyzed by considering the following two independent cases:

Case 1. $Q = Q'$: The overlap of Q and Q' is $s, s = |D|$, and there are n occurrences for such a case.

Case 2. $Q \neq Q'$: The overlap of Q and Q' is λ and there are $\binom{n}{2}$ occurrences for such a case.

Thus, the EQOS of \mathcal{Q} is

$$\frac{s \cdot n + \lambda \cdot \binom{n}{2}}{\binom{n+1}{2}} = \frac{2s + \lambda \cdot (n - 1)}{n + 1}$$

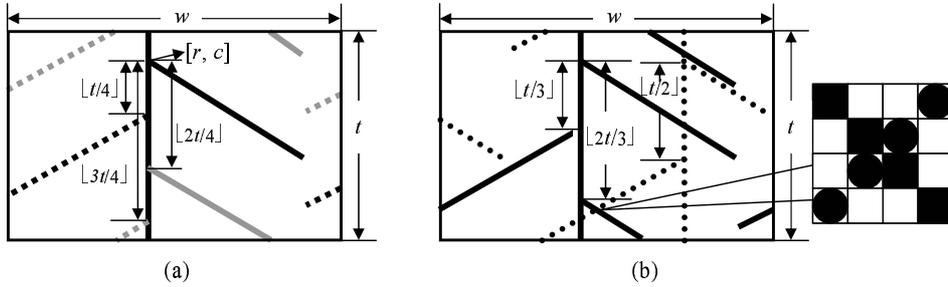


Fig. 2. (a) the “Christmas tree” structure of an e-torus(4) quorum, and (b) the overlap of an e-torus(2) quorum and an e-torus(3) quorum.

3.5 The EQOS of FPP Quorum Systems

The *finite projective plane (FPP) quorum system* [13] arranges elements of the universal set $U = \{0, \dots, n - 1\}$ as vertices on a hypergraph called the *finite projective plane*, which has n vertices and n edges, such that each edge is connected to k vertices and two edges have exactly one common vertex. (Note that the hypergraph is a generalization of typical graphs, in which each edge is connected to exactly two vertices.) A quorum can be formed by the set of all vertices connected by the edge, and thus has a size of k . It has been shown in [13] that a FPP can be constructed when $n = s(s - 1) + 1$ and $s - 1$ is a prime power. Otherwise, the FPP may or may not exist. In [14], the FPP construction is associated to the construction of Singer difference sets, and it is shown that the FPP quorum system can be regarded as a special case of the cyclic quorum system when $n = s(s - 1) + 1$ and $s - 1$ is a prime power. We can also observe that the FPP quorum system is a special case of cyclic quorum systems that is constructed on the basis of a 1-difference set. Thus,

the EQOS of the FPP quorum system is

$$\frac{s \cdot n + \binom{n}{2}}{\binom{n+1}{2}} = \frac{2s + n - 1}{n + 1}$$

	C_0	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	
					T			T'			}
					T	R	R	T'R	R	R	
					T	R	R	T'R	R	R	}
k_l	L	L	L	L	T			T'			
	R'	R'	R'		T			T'	R'	R'	}
k'_l				L'	L'T	L'	L'	T'			
	}			}		}		}			
	d			$w_l - d$		$d - 1$		$w_r - d$			

Legend of elements:

T: in trunk of Q ; R: in right branches of Q ; L: in left branches of Q
T': in trunk of Q' ; R': in right branches of Q' ; L': in left branches of Q'

Fig. 3. One possibility of a ge-torus(k) quorum and a ge-torus(k') quorum of distance d .

3.6 The EQOS of Extended Torus Quorum Systems

The *extended torus* (*e-torus*) quorum system is basically an extension of the torus quorum system. Similar to the torus quorum system, it is also defined on the basis of two given integers t and w such that $U = \{0, 1, \dots, tw - 1\}$ is the universal set. Elements of U are arranged in a $t \times w$ array. Below, we use $[x, y]$ as an array index, $0 \leq x < t$ and $0 \leq y < w$.

Definition 9 On a $t \times w$ array, a positive half diagonal starting from position $[x, y]$, where $0 \leq x < t$ and $0 \leq y < w$, consists of element $[x, y]$ plus $\lfloor w/2 \rfloor$ elements $[(x + i) \bmod t, (y + i) \bmod w]$, for $i = 1.. \lfloor w/2 \rfloor$. A negative half diagonal starting from position $[x, y]$ consists of element $[x, y]$ plus $\lceil w/2 \rceil - 1$ elements $[(x + i) \bmod t, (y - i) \bmod w]$, for $i = 1.. \lceil w/2 \rceil - 1$.

Intuitively, a positive (resp., negative) half diagonal is a partial diagonal on the array starting from the array index $[x, y]$ with a length $\lfloor w/2 \rfloor + 1$ (resp., $\lceil w/2 \rceil$). A positive diagonal goes in the southeast direction, while a negative one goes in the southwest direction. The diagonal is slightly different from typical “diagonal” in matrix algebra in that the array is not necessarily square and that the torus has the wrap-around property.

Definition 10 Given any integer $k \leq t$, a quorum of an *e-torus*(k) quorum system is formed by picking any position $[r, c]$, where $0 \leq r < t$ and $0 \leq c < w$, such that the quorum contains all elements on column c plus k half diagonals.

These k half diagonals alternate between positive and negative ones, and start from the following positions:

$$\left[r + \left\lfloor i \times \frac{t}{k} \right\rfloor, c \right], \quad i = 0..k - 1.$$

Each quorum in the e-torus(k) quorum system looks like a Christmas tree with a trunk in the middle and k branches, each as a half diagonal, alternating between positive and negative ones. Fig. 2(a) illustrates the conceptual structure of an e-torus(4) quorum. It is shown in [10], the e-torus quorum system satisfies the rotation closure property. To be more precise, if G is an e-torus(k_1) quorum and H is an e-torus(k_2) quorum derived from the same array, for any integers i and j , we have $|\text{rotate}(G, i) \cap \text{rotate}(H, j)| \geq \lfloor (k_1 + k_2)/2 \rfloor$.

The e-torus quorum system is used as foundation of the following adaptive quorum-based power saving algorithm proposed in [10]. A node is assumed to be able to calculate its mobility, which is ranked into k levels, where level 1 means the lowest mobility, and level k means the highest mobility. Whenever a node determines that its mobility falls within level i ($1 \leq i \leq k$), it adjusts its quorum intervals based on any e-torus(i) quorum. Consequently, a node can dynamically adjust its sensibility to the environment change in its neighborhood.

Below, we analyze the EQOS for a more general form of e-torus quorum systems, the *ge-torus*(k), as defined in Def. 11. We believe that the EQOS of the *ge-torus*(k) quorum system can be used as an estimation of EQOS of the e-torus(k) quorum system since the latter is a special case of the former and they are all based on randomly choosing elements.

Definition 11 *Given a universal set, in which elements are arranged as a $t \times w$ array with the rightmost column being regarded as wrapping around back to the leftmost column, a quorum of a *ge-torus*(k) quorum system (k is an integer less than t) is formed by picking all elements of any column c , $0 \leq c \leq w - 1$, plus $\lceil k/2 \rceil \cdot \lfloor w/2 \rfloor$ elements, $\lceil k/2 \rceil$ of which fall in column $c + i$, $i = 1.. \lfloor w/2 \rfloor$, and plus $\lfloor k/2 \rfloor \cdot (\lceil w/2 \rceil - 1)$ elements, $\lfloor k/2 \rfloor$ of which fall in column $c - i$, $i = 1.. \lceil w/2 \rceil - 1$.*

Suppose Q is a quorum in the *ge-torus*(k) quorum system and Q' is a quorum in the *ge-torus*(k') quorum system, where Q contains all elements of column c and Q' contains all elements of column c' . We define d , the distance of c and c' , to be $\text{Min}((c - c') \bmod w, (c' - c) \bmod w)$. The EQOS of the *ge-torus* quorum system can be figured out by considering the following five independent cases of Q and Q' . It is noted that in the following analysis, we let $E(h, i, j)$, $i, j \leq h$, be the expected number of the common members of two independently chosen subsets of a set of h elements, where the first

and the second subsets respectively contain i and j elements (without loss of generality, we can assume $i \leq j$). By the knowledge of combinatorics, we can derive that $E(h, i, j) = i \frac{\binom{h-1}{j-1}}{\binom{h}{j}}$. It is also noted that we let $k_r = \lceil k/2 \rceil$, $k_l = \lfloor k/2 \rfloor$, $k'_r = \lceil k'/2 \rceil$, $k'_l = \lfloor k'/2 \rfloor$, $w_r = \lfloor w/2 \rfloor$ and $w_l = \lceil w/2 \rceil - 1$.

Case 1. $d = 0$ (i.e., $c = c'$): The overlap of Q and Q' is expected to have size $s_1 = t + E(t, k_r, k'_r) \cdot w_r + E(t, k_l, k'_l) \cdot w_l$ and there are w occurrences for such a case.

Case 2. $1 \leq d < w_r$ and $((c' - c) \bmod w) = d$: The overlap of Q and Q' is expected to have size $s_2(d) = (d - 1) \cdot E(t, k_r, k'_l) + (w_r - d) \cdot E(t, k_r, k'_r) + d \cdot E(t, k_l, k'_r) + (w_l - d) \cdot E(t, k_l, k'_l) + k'_l + k_r$ and there are w occurrences for such a case for each $d, 1 \leq d < w_r$. Please refer to Fig. 3 for an illustration of this case.

Case 3. $1 \leq d < w_r$ and $((c - c') \bmod w) = d$: This case is opposite to Case 2. The overlap of Q and Q' is expected to have size $s_3(d) = (d - 1) \cdot E(t, k'_r, k_l) + (w_r - d) \cdot E(t, k'_r, k_r) + d \cdot E(t, k'_l, k_r) + (w_l - d) \cdot E(t, k'_l, k_l) + k_l + k'_r$ and there are w occurrences for such a case for each $d, 1 \leq d < w_r$.

Case 4. $d = w_r$ and w is even: The overlap of Q and Q' has size $s_4 = (w_r - 1) \cdot (E(t, k_r, k'_l) + E(t, k_l, k'_r)) + k_r + k'_r$ and there are w occurrences for such a case.

Case 5. $d = w_r$ and w is odd: The overlap of Q and Q' may have size $s_{51} = w_r \cdot E(t, k'_r, k_l) + (w_r - 1) \cdot E(t, k_r, k'_l) + k_r + k'_l$ and there are w occurrences for such a possibility. Moreover, the overlap of Q and Q' may have size $s_{52} = w_r \cdot E(t, k_r, k'_l) + (w_r - 1) \cdot E(t, k'_r, k_l) + k'_r + k_l$ and there are w occurrences for such a possibility.

There are w^2 possible permutations of Q and Q' . Thus, the EQOS of the ge-torus quorum system under $\{0, \dots, n-1\}$ is

$$\begin{aligned} & \frac{w \cdot s_1 + w \cdot \sum_{d=1}^{w_r-1} [s_2(d) + s_3(d)] + w \cdot s_4}{w^2} \\ &= \frac{s_1 + \sum_{d=1}^{w_r-1} [s_2(d) + s_3(d)] + s_4}{w}, \text{ for } w \text{ is even,} \\ & \text{or } \frac{w \cdot s_1 + w \cdot \sum_{d=1}^{w_r-1} [s_2(d) + s_3(d)] + w \cdot (s_{51} + s_{52})}{w^2} \\ &= \frac{s_1 + \sum_{d=1}^{w_r-1} [s_2(d) + s_3(d)] + s_{51} + s_{52}}{w}, \text{ for } w \text{ is odd.} \end{aligned}$$

Table 1

EQOS, MQOS and SQOS comparisons for optimal and near optimal quorum systems

Quorum system	EQOS	MQOS	SQOS
grid	$\frac{4n-4\sqrt{n}+1}{n}$	$(n - \sqrt{n} + 1)$	2
torus	2	n	1
cyclic	$\frac{2s+\lambda \cdot (n-1)}{n+1}$, where s is the quorum size	n	1
FPP	$\frac{2s+n-1}{n+1}$, where s is the quorum size, $s(s-1)+1=n$, and $s-1$ is a prime power	n	1
ge-torus(k), (under $t \times w$ array arrangement)	<ul style="list-style-type: none"> • For w is even: $\frac{s_1 + \sum_{d=1}^{w-1} [s_2(d) + s_3(d)] + s_4}{w}$ • For w is odd: $\frac{s_1 + \sum_{d=1}^{w-1} [s_2(d) + s_3(d)] + s_{51} + s_{52}}{w}$ <p>(please refer to Section 3.6 for the meaning of $s_1, s_2(d), s_3(d), s_4, s_{51}$, and s_{52})</p>	For e-torus(k_1) and e-torus(k_2): <ul style="list-style-type: none"> • n, for $(k_1 = k_2 = 1)$, $(k_1 = 2 \wedge k_2 = 1)$, or $(k_1 = 1 \wedge k_2 = 2)$ • $(n - 1)$, for $(k_1 = 3 \wedge k_2 = 1)$ or $(k_1 = 1 \wedge k_2 = 3)$ • $n - \lfloor 2t/k_1 \rfloor$, for $(k_1 = 1 \wedge k_2 = 4)$ or $(k_1 = 4 \wedge k_2 = 1)$ • less than $(n - \lfloor 2t/k_1 \rfloor)$, for $(k_1 > 4 \wedge k_2 = 1)$ or $(k_1 = 1 \wedge k_2 > 4)$ • $(n - w + 1)$, for $(k_1 = 2 \wedge k_2 = 2)$ • less than $(n - w + 1)$, for $(k_1 \geq 2 \wedge k_2 > 2)$ or $(k_1 > 2 \wedge k_2 \geq 2)$ 	For e-torus(k_1) and e-torus(k_2): $\lfloor (k_1 + k_2)/2 \rfloor$

4 Analysis and Simulation Result Comparisons

Table 1 summarizes the EQOS, MQOS and SQOS analysis results for the quorum systems satisfying the rotation closure property and having optimal or near optimal quorum sizes. We can observe that the FPP, the cyclic, the torus quorum systems have the same MQOS and SQOS. We are hard to judge them by MQOS and SQOS. However, all quorum systems have different EQOS. We can then choose proper quorum systems on the basis of EQOS.

Fig. 4 (a), (b) and (c) further demonstrate the EQOS, the active ratio QS/RS, and the ratio EQOS/QS of different quorum systems, where QS stands for the quorum size and RS stands for the round size n , $n = 5..100$. It is noted that in Fig. 4, ge-torus(k_1 vs. k_2) in the legend stands for the case using a pair of quorums in the quorum systems ge-torus(k_1) and ge-torus(k_2). By observing the figure, we find that ge-torus(3) quorum system (the case of ge-torus(3 vs. 3)) has the largest EQOS (the best neighbor sensibility in the average case) and the highest active ratio (the most energy consumption). On the contrary, the FPP quorum system has the smallest EQOS and the lowest active ratio. However, by the EQOS/QS measurement, the ge-torus(3) quorum system outperforms the FPP quorum system. The EQOS/QS can be used as an estimation of how much a member (an active interval) of a quorum contributes to the EQOS value. It should thus be as large as possible. In summary, we suggest adopting adaptive power-saving algorithm using ge-torus(k) quorum system because ge-torus(k) quorum system may have the highest neighbor sensibility, comparably low active ratio (note that a ge-torus(1) is actually a

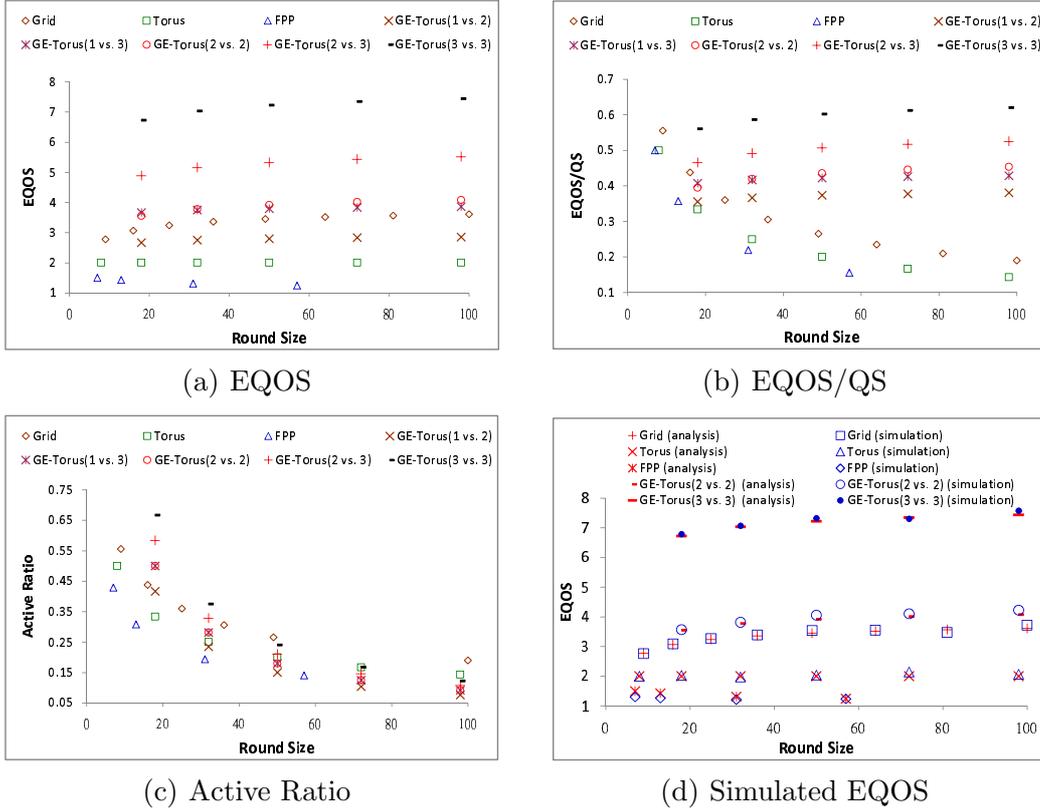


Fig. 4. Comparisons of different quorum systems in terms of EQOS, EQOS/QS, Active Ratio and Simulated EQOS for round size $n=5..100$

torus quorum system whose active ration is near optimal), and the highest EQOS/QS value. It is noted that larger k leads to larger EQOS and thus better neighbor sensibility. This is because larger k implies a larger quorum size, which in turn keeps a node active for more beacon intervals to receive neighbors' beacons. Certainly, we should adjust k to be a proper value according to the network environment parameters, such as the node mobility, and/or the EQOS requirement, etc.

We also write a simulator in Java for performing simulation experiments to derive EQOS for power-saving algorithms using optimal or near optimal quorum systems. The simulation assumes a 1000m by 1000m area in which 100 nodes move according to the random way point model. To be more precise, every node moves at a random speed between 0 and 10m/s toward a randomly chosen destination. At arriving the destination, the node pauses for a fixed period of 20 seconds and then move toward the next destination. Each node has a radio communication range of 250m and can turn the radio into active or power-saving modes. The beacon interval has a length of 100ms, and every node sends a beacon at the beginning of the quorum beacon interval. Each simulation experiment lasts for 1000 seconds. The analysis and simulation results are shown together in Fig. 4(d). We can see that the two sets of results

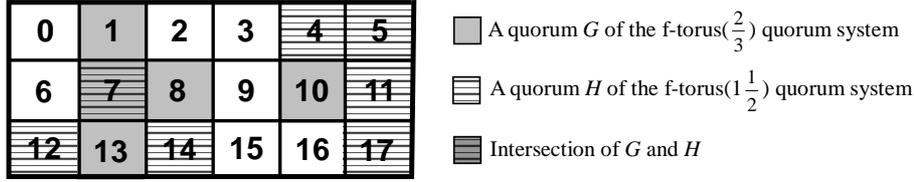


Fig. 5. Two quorums of the f -torus quorum system in a 3×6 torus.

coincide very closely.

5 The Fraction Torus Quorum System and Its EQOS

By the EQOS analysis and simulation shown above, we observe that we can further shrink the quorum sizes to save energy if we desire a node to receive in average one beacon from every neighboring node. In view of this, we propose the concept of the *fraction torus (f -torus) quorum system*, as defined below.

Definition 12 *Given a universal set, in which elements are arranged as a $t \times w$ array with the rightmost column being regarded as wrapping around back to the leftmost column, a quorum of an f -torus($k+f$) quorum system is formed by picking*

- (a) *all elements of a column c , $0 \leq c \leq w - 1$*
- (b) *$\lceil k/2 \rceil \cdot \lceil w/2 \rceil$ elements, $\lceil k/2 \rceil$ of which fall in column $c + i$, $i = 1.. \lceil w/2 \rceil$*
- (c) *$\lfloor k/2 \rfloor \cdot (\lceil w/2 \rceil - 1)$ elements, $\lfloor k/2 \rfloor$ of which fall in column $c - i$, $i = 1.. \lceil w/2 \rceil - 1$*
- (d) *$f \cdot \lceil w/2 \rceil$ elements, each of which falls in columns $c + 1, \dots$, or $c + \lceil w/2 \rceil$ if k is even (or $f \cdot (\lceil w/2 \rceil - 1)$ elements, each of which falls in columns $c - 1, \dots$, or $c - (\lceil w/2 \rceil - 1)$ if k is odd).*

Note that k ($0 \leq k \leq t$) is an integer and f is a fraction of the value $\frac{i}{\lceil w/2 \rceil}$, $i = 0, \dots$, or $\lceil w/2 \rceil - 1$ if k is even (f is of the value $\frac{i}{\lceil w/2 \rceil - 1}$, $i = 0, \dots$, or $\lceil w/2 \rceil - 2$ if k is odd).

Fig. 5 illustrates the construction of two f -torus quorums G and H under $U = \{0, \dots, 17\}$ with $t = 3$ and $w = 6$. G belongs to an f -torus($\frac{2}{3}$) quorum system and is formed by picking all the elements of the second column plus two more ($= \frac{2}{3} \cdot \lfloor \frac{6}{2} \rfloor$) elements, one from the third and one from the fifth columns. H belongs to an f -torus($1\frac{1}{2}$) quorum system and is formed by picking all the elements of the sixth column plus three more elements, from the first, second, and third columns, respectively, plus one more ($= \frac{1}{2} \cdot (\lfloor \frac{6}{2} \rfloor - 1)$) element from the fifth column. Note that G and H intersect at element 7.

Suppose Q is a quorum in the f -torus($k+f$) quorum system and Q' is a quorum in the f -torus($k'+f'$) quorum system, where Q contains all elements of column c and Q' contains all elements of column c' . Below, we analyze the EQOS of

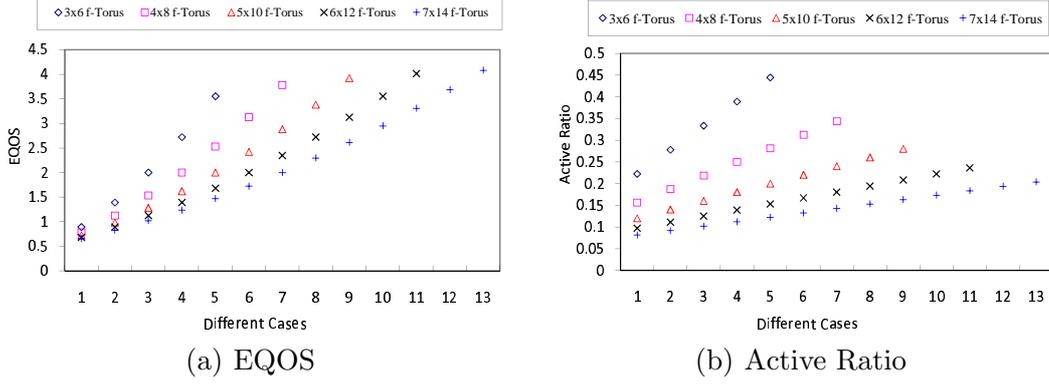


Fig. 6. Comparisons of different cases of f-torus quorum systems in terms of EQOS and active ratio

the f-torus quorum system. It is noted that in the following analysis we obey the notations used in Section 3.6. We also let $f_r = f$ and $f_l = 0$ if k is even, let $f_r = 0$ and $f_l = f$ if k is odd, let $f'_r = f'$ and $f'_l = 0$ if k' is even, and let $f'_r = 0$ and $f'_l = f'$ if k' is odd. Moreover, we use $E_f(h, i, j, p, q)$, where i and j are integers and p and q are fractions, to represent the expected number of the common members of two independently chosen subsets of a set of h elements. The integers i and j stand for the first and the second subset cardinalities, respectively. And the fractions p and q , $0 \leq p, q < 1$, respectively represent the probabilities to add one more element in the first subset and the second subset. We can derive that $E_f(h, i, j, p, q) = E(h, i, j) + \frac{i}{h} \cdot q + \frac{j}{h} \cdot p + \frac{1}{h} \cdot p \cdot q$

In a manner similar to that of the ge-torus quorum system, we can figure out the EQOS of the f-torus quorum system by considering the following five independent cases.

Case 1. $d = 0$ (i.e., $c = c'$): The overlap of Q and Q' is expected to have size $s_1 = t + E_f(t, k_r, k'_l, f_r, f'_l) \cdot w_r + E(t, k_l, k'_l, f_l, f'_l) \cdot w_l$ and there are w occurrences for such a case.

Case 2. $1 \leq d < w_r$ and $((c' - c) \bmod w) = d$: The overlap of Q and Q' is expected to have size $s_2(d) = (d - 1) \cdot E_f(t, k_r, k'_l, f_r, f'_l) + (w_r - d) \cdot E_f(t, k_r, k'_r, f_r, f'_r) + d \cdot E_f(t, k_l, k'_r, f_l, f'_r) + (w_l - d) \cdot E_f(t, k_l, k'_l, f_l, f'_l) + k'_l + k_r + f'_l + f_r$ and there are w occurrences for such a case for each d , $1 \leq d < w_r$.

Case 3. $1 \leq d < w_r$ and $((c - c') \bmod w) = d$: This case is opposite to Case 2. The overlap of Q and Q' is expected to have size $s_3(d) = (d - 1) \cdot E_f(t, k'_r, k_l, f'_r, f_l) + (w_r - d) \cdot E_f(t, k'_r, k_r, f'_r, f_r) + d \cdot E_f(t, k'_l, k_r, f'_l, f_r) + (w_l - d) \cdot E_f(t, k'_l, k_l, f'_l, f_l) + k_l + k'_r + f_l + f'_r$ and there are w occurrences for such a case for each d , $1 \leq d < w_r$.

Case 4. $d = w_r$ and w is even: The overlap of Q and Q' has size $s_4 = (w_r - 1) \cdot (E_f(t, k_r, k'_l, f_r, f'_l) + E_f(t, k_l, k'_r, f_l, f'_r)) + k_r + k'_r + f_r + f'_r$ and there are w

occurrences for such a case.

Case 5. $d = w_r$ and w is odd: The overlap of Q and Q' may have size $s_{51} = w_r \cdot E_f(t, k'_r, k_l, f'_r, f_l) + (w_r - 1) \cdot E_f(t, k_r, k'_l, f_r, f'_l) + k_r + k'_l + f_r + f'_l$ and there are w occurrences for such a possibility. Moreover, the overlap of Q and Q' may have size $s_{52} = w_r \cdot E_f(t, k_r, k'_l, f_r, f'_l) + (w_r - 1) \cdot E_f(t, k'_r, k_l, f'_r, f_l) + k'_r + k_l + f'_r + f_l$ and there are w occurrences for such a possibility.

There are w^2 possible permutations of Q and Q' . Thus, the EQOS of the f -torus quorum system under $\{0, \dots, n-1\}$ is

$$\begin{aligned} & \frac{w \cdot s_1 + w \cdot \sum_{d=1}^{w_r-1} [s_2(d) + s_3(d)] + w \cdot s_4}{w^2} \\ &= \frac{s_1 + \sum_{d=1}^{w_r-1} [s_2(d) + s_3(d)] + s_4}{w}, \text{ for } w \text{ is even,} \\ \text{or } & \frac{w \cdot s_1 + w \cdot \sum_{d=1}^{w_r-1} [s_2(d) + s_3(d)] + w \cdot (s_{51} + s_{52})}{w^2} \\ &= \frac{s_1 + \sum_{d=1}^{w_r-1} [s_2(d) + s_3(d)] + s_{51} + s_{52}}{w}, \text{ for } w \text{ is odd.} \end{aligned}$$

Fig. 6 (a) and (b) demonstrate the EQOS and the active ratio (QS/RS) comparisons for different cases of f -torus quorum systems. Under a $t \times w$ torus, we consider the cases of f -torus($\frac{1}{\lfloor w/2 \rfloor}$), ..., f -torus($\frac{\lfloor w/2 \rfloor - 1}{\lfloor w/2 \rfloor}$), f -torus(1), f -torus($1 + \frac{1}{\lfloor w/2 \rfloor - 1}$), ..., f -torus($1 + \frac{\lfloor w/2 \rfloor - 2}{\lfloor w/2 \rfloor - 1}$), and f -torus(2). For example, under a 7×14 torus, we consider the following 13 cases: f -torus($\frac{1}{7}$), f -torus($\frac{2}{7}$), f -torus($\frac{3}{7}$), f -torus($\frac{4}{7}$), f -torus($\frac{5}{7}$), f -torus($\frac{6}{7}$), f -torus(1), f -torus($1\frac{1}{6}$), f -torus($1\frac{2}{6}$), f -torus($1\frac{3}{6}$), f -torus($1\frac{4}{6}$), f -torus($1\frac{5}{6}$) and f -torus(2). By Fig. 6, we observe that a larger $(t \times w)$ value leads to lower EQOS and active ratio. This is because the round size is proportional to the $(t \times w)$ value, while the quorum size is proportional to the $(t + w)$ value. When t and w grow, the $(t \times w)$ value grows much faster than the $(t + w)$ value. The active ratio thus decreases since it equals to the ratio of the quorum size over the round size. It is noted that EQOS has a similar condition. By Fig. 6, we also observe that we can achieve very low active ratios while keeping the EQOS larger than 1. For example, the f -torus($\frac{3}{7}$) quorum system has the EQOS of 1.020408163, and has a very low active ratio of 0.102040816.

By many possible values of the integer k and the fraction f , the f -torus($k + f$) quorum system provides much flexibility for us to design adaptive quorum-based asynchronous power saving algorithms. The algorithms can now rank a node's mobility into much more levels and thus are more adaptive to mobility changes than those using e -torus quorum systems.

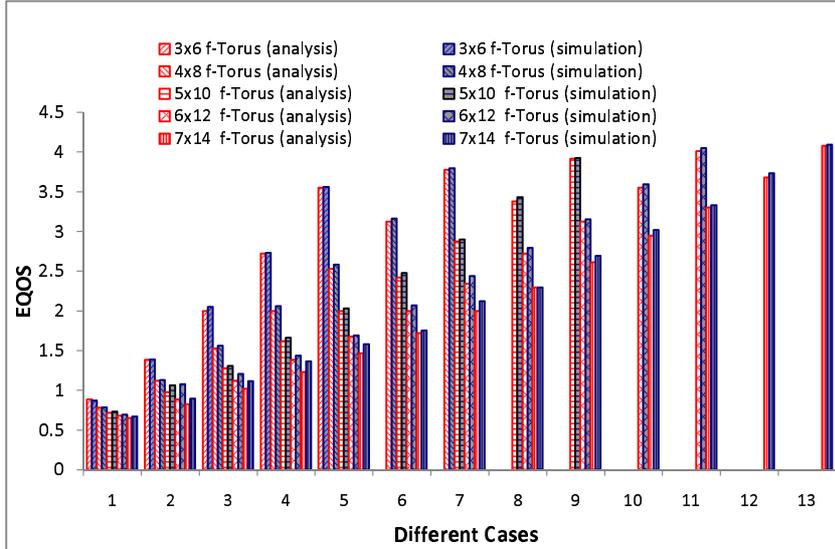


Fig. 7. Analysis and simulation results of EQOS for different cases of f-torus quorum systems

The f-torus quorum system is a generalization of the e-torus quorum system. For $k \geq 1$ and $f = 0$, the f-torus($k + f$) quorum system is actually the ge-torus(k) quorum system. For example, f-torus(1) and f-torus(2) are ge-torus(1) and ge-torus(2) quorum systems. We can check that the EQOS analysis equations of f-torus and ge-torus quorum systems are the same when we substitute f by 0 in the equations. By the simulation results in Fig. 8, we can see that the f-torus(k) and ge-torus(k) quorum systems, $k=1$ or 2, have equal EQOS values if we neglect the small statistical errors. In general, there are more possibilities of f-torus quorum systems than ge-tours quorum systems. Furthermore, many f-torus quorum systems are of smaller active ratios than ge-torus quorum systems under the same torus structure. For example, under the 7×14 torus, f-torus($\frac{1}{7}$), f-torus($\frac{2}{7}$), f-torus($\frac{3}{7}$), f-torus($\frac{4}{7}$), f-torus($\frac{5}{7}$) and f-torus($\frac{6}{7}$) all have smaller active ratios than ge-torus(1). This demonstrates the flexibility and the efficiency of f-torus quorum systems.

6 Conclusion

In this paper, we have proposed the concept of the expected quorum overlap size (EQOS) for evaluating the average-case neighbor sensibility of a quorum-based asynchronous power saving algorithm in IEEE 802.11 MANETs. We have analyzed the EQOS for the FPP, grid, cyclic, torus and e-torus quorum systems that satisfy the rotation closure property and have optimal or near optimal quorum sizes (or active ratios). As we have shown, with the help of the EQOS, we can properly select quorum systems to construct asynchronous power saving algorithms for specific environments. Observing that the optimal

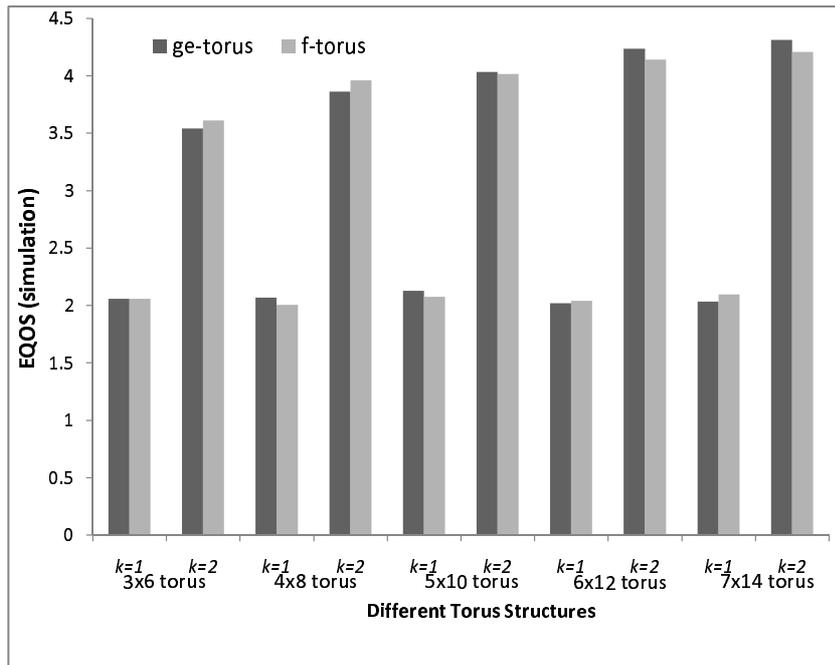


Fig. 8. EQOS simulation results for ge-torus(k) and f-torus(k) quorum systems, $k = 1$ or 2 , under different torus structures

and near optimal quorum systems have EQOS values much larger than one, we are inspired to devise a new quorum system, called the fraction torus (f-torus) quorum system, for the construction of mobility-adaptive power saving algorithms. The f-torus quorum systems can further reduce the energy expenditure by shrinking the quorum size, while keeping the EQOS larger than one so that a node is expected to receive one or more beacons from a neighboring node in a round. By many possible values of the integer k and the fraction f , the f-torus($k + f$) quorum system provides much flexibility for us to design adaptive quorum-based asynchronous power saving algorithms. We have performed simulation experiments to derive EQOS values for the optimal, near optimal and f-torus quorum systems. As we have shown, the analysis and simulation results of EQOS coincide very closely.

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