國立中央大學資訊工程學系 112 學年度第二學期博士班資格考試題紙

科目： **演 算 法** (Algorithms) 第一頁 共一頁(page 1 of 1)

1. The following statements about minimum spanning tree (MST) may or may not be correct. Assume that the weighted graph $G = (V, E)$ is undirected and connected. Do not assume that edge weights are distinct unless this is specifically stated. For each of the following statements either prove it is correct or give a counterexample to show that it is incorrect.
   (A) If $|E| \geq |V|-1$, and there is a unique heaviest edge, then this edge cannot be part of any MST of G. (5%)
   (B) If G has a cycle with a unique heaviest edge e, then e cannot be part of any MST of G. (5%)
   (C) If the lightest edge in G is unique, then it must be part of every MST of G. (5%)
   (D) If G has a cycle with a unique lightest edge e, then e must be part of some MST of G. (5%)
   (E) The shortest path between two nodes is necessarily part of some MST of G. (5%)

2. Give an asymptotically tight bound for each of the following functions. You may assume that the function value of each function is constant for $n \leq 2$. (25%)
   $t(n) = n + \sum_{k=1}^{n-1}[t(k) + t(n-k)]$, $f(n) = 2f(n/4) + \sqrt{n}$,
   $g(n) = g(n/2) + \sqrt{n}$, $h(n) = 5h(n/2) + (n \lg n)^2$,
   $a(n) = a(n-1) + n^k \lg n$.

3. **Recall** the **candyman can problem**: *A person has some candies that are of different weight and wants to divide them evenly between 3 children, so that they don't get jealous at each other. He wants to know how bad the fairest distribution is, i.e. what is the minimum difference in the candies weight, of the kid getting the candies of the most total weight and the kid getting the least. For example, assume that he gives the three children candies of weight $a \geq b \geq c$, respectively. The badness of this distribution is $a - c$.*

   This problem: Give a decision version of the candyman can problem, and show that it is an NP and NP-hard problem. (25%)

4. Give a linear-time algorithm that takes as input a tree and determines whether it has a perfect matching: a set of edges that touches each node exactly once? (25%)